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Threshold-initiated spatial public goods games

Weijie Wang [a,1], Zhehang Xu [a,1], Shijia Hua [b], Longqing Cui [a,*], Jianlin Zhang [a,*], Fanyuan Meng [a,*]

[a] *Alibaba Research Center for Complexity Sciences, Hangzhou Normal University, Hangzhou, 311121, Zhejiang, China*
[b] *College of Science, Northwest A & F University, Yangling, 712100, China*

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ABSTRACT

The Public Goods Game (PGG) encounters hurdles when donations are scarce, resulting in failed game initiations. To investigate such phenomena, we propose a threshold-based spatial PGG model operating on a lattice with periodic boundaries. Players strategically choose between cooperation (C) and defection (D), with PGG initiation determined by a cooperative contribution threshold. Additionally, we introduce an adjustment factor for player reputation, reflecting how individual strategy choices and responses from interacting partners influence reputation changes. We hypothesize that within non-initiated PGG, defectors' reputations decrease, while within initiated PGG, defectors are evaluated and penalized by cooperators. Our findings reveal that higher initiation thresholds can enhance final cooperation levels. Moreover, the inclusion of the reputation adjustment factor acts as a catalyst for cooperative behavior. Interestingly, greater uncertainty in strategy adoption is associated with increased cooperative levels under higher initiation thresholds. This study adds new insights into the evolution of cooperation in the context of spatial structure.

1. Introduction

Cooperation is ubiquitous in human society, and explaining how sustained cooperation emerges among selfish individuals has long been an interdisciplinary hot topic [1–3]. Evolutionary game theory provides a powerful theoretical framework for understanding this issue [4,5]. Many classic game models have been introduced into the evolutionary game framework to describe cooperation dilemmas in different environments, such as the Prisoner's Dilemma Game (PDG) [6,7], the Snowdrift Game (SDG) [8–10], and the Public Goods Game (PGG) [11–13]. Among them, the public goods game model is widely used to describe cooperation dilemmas in multi-agent interactions in real-world systems, such as public transportation [14] and environmental issues [15]. In the traditional public goods game, each individual in a group faces the choice of whether to contribute to a common pool. Those who contribute are called cooperators, while those who do not contribute are called defectors. Eventually, the total contribution of all individuals is multiplied by a coefficient and then evenly distributed among all group members. Although the maximum benefit for the group occurs when everyone cooperates, defectors can reap the same benefits as cooperators without incurring the additional costs associated with cooperation, leading to the Nash Equilibrium of all individuals choosing to defect, known as the "tragedy of the commons" [16,17].

To explain the emergence of cooperative behavior, scholars have proposed various mechanisms. Nowak summarized the five major rules of cooperation evolution, including kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection [18]. Related studies have found that within the network reciprocity, the factors such as reward and adjustment mechanisms [19–22], environment [23,24], voluntary participation mechanisms [25–27], memory mechanisms [28–30], heterogeneous investment [31–33], and coevolutionary mechanisms [34–37] all have a significant impact on the evolution of cooperation in a system.

Reputation, a measure of an individual's past behavior, is another critical factor influencing the evolution of cooperation [38–40]. In human societies, reputation serves as a powerful mechanism for promoting cooperation and punishing defection. Players may formulate their decisions or strategies predicated on the reputations of others, which adds a layer of complexity, fostering a richer and more dynamic model of interaction. The integration of reputation into PGG models has provided deeper insights into the evolution of cooperation, showing how reputation-based strategies can sustain cooperation in scenarios where other mechanisms fail.

In real-life scenarios, certain levels of participation or contribution are often required to initiate or sustain public goods. This concept

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[*] Corresponding authors.
E-mail addresses: cuilongqing@hznu.edu.cn (L. Cui), jianlinzhang@hznu.edu.cn (J. Zhang), fanyuan.meng@hotmail.com (F. Meng).
[1] These two authors contribute equally.

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is encapsulated in threshold mechanisms in PGG, where a minimum number of cooperators is needed to produce a public good. Thresholds introduce a more nuanced and realistic layer to PGG, capturing scenarios like community projects, where insufficient participation leads to failure [41–44]. The introduction of thresholds in PGG models has shown to significantly affect the emergence and stability of cooperative behavior, with outcomes highly sensitive to the heterogeneity and size of these thresholds.

In the study of [45], all the endowments will vanish in the PGG within each group with a certain probability, determined by the variance between the total contributions gathered and a predefined collective target. However, in practical scenarios, the initiation of the public goods game may not occur if contributions fail to meet the collective target, and consequently, no players obtain or suffer any loss in payoff. Additionally, the increased endowment in the group of reaching the target for cooperators or defectors may be related with the number of contributions in the group. Therefore, we introduce a novel model called the threshold-initiated Public Goods Game within a spatial lattice framework with periodic boundaries. We further incorporates reputation adjustment mechanisms into the model. This innovative model enables us illuminate the complex interplay between spatial structure, threshold effects, and reputation mechanisms in shaping cooperative levels in spatial structures.

The remainder of the paper is organized as follows: Section 2 details the model and its underlying assumptions. Section 3 presents the results of our simulations and analyses. Section 4 concludes with a summary of our findings and suggestions for future research.

2. The model

Our model unfolds within a spatial lattice with periodic boundaries, which consists of $N = L \times L$ players, denoted as $i \in \{1, 2, \ldots, N\}$. Each player occupies a grid intersection, surrounded by four nearest neighbors. At the initial time step $t = 0$, players randomly choose a strategy of cooperation (C) or defection (D) with equal probability.

2.1. Public goods game

In our spatial Public Goods Game (PGG), each player interacts with their four nearest neighbors, forming a group g of $z = 5$ participants. The payoff for player i within each group is defined as

$$p_i(g) = \begin{cases} \frac{R\sum_{j\in g} s_j}{z}, & s_i = 0, \\ \frac{R\sum_{j\in g} s_j}{z} - 1, & s_i = 1. \end{cases} \tag{1}$$

Here, $s_i = 1$ and $s_i = 0$ represent the cooperation or defection choice of player i, respectively. If $s_i = 0$, the payoff for player i is a fraction of the total contributions from all group members, scaled by the synergy factor $R \geq 1$. The word "synergy" generally refers to the combined or cooperative effects that are greater than the sum of individual effects. In this context, R is a parameter that quantifies the level of synergy in the group interaction. On the other hand, if $s_i = 1$, the payoff of player i has an additional cost of 1.

2.2. Threshold

All PGG unfold simultaneously at the subsequent time step $t = 1, 2, \ldots$. Each player has the opportunity to engage in five groups ($Z = 5$): one centered on themselves, and the others involving each of their nearest neighbors (see Fig. 1).

Some works emphasize the necessity of a minimal number of cooperators to initiate the PGG [46–48]. To capture this phenomenon, we introduce an integer threshold ϕ, wherein PGG initiation in a specific group g occurs only if the collective contributions within the group exceeds or equals to ϕ (i.e., $\sum_{j\in g} s_j \geq \phi$). Consequently, the five groups are categorized into two subsets: initiated (G_1) and non-initiated (G_0) groups within our model. It is assumed that players derive no payoff from non-initiated (G_0) groups. The total payoff for each player i is then the sum of payoffs from all initiated PGG groups:

$$P_i = \sum_{j\in g\in G_1} p_j(g). \tag{2}$$

2.3. Reputation

At the outset of the interaction, each player begins with a reputation of $r = 1$. As the interaction unfolds over the discrete time step t, the reputation of each player i involved in initiated (G_1) groups undergoes a dynamic update process. This process is governed by a rule that dictates how the reputation of an individual changes based on their actions and the actions of their interaction partners.

The update process is delineated as follows:

$$r_i(t+1) = \begin{cases} r_i(t) - \delta, & s_i(t) = 0, s_j(t) = 1, \\ r_i(t), & s_i(t) = s_j(t), \\ r_i(t) + \delta, & s_i(t) = 1, s_j(t) = 0, \end{cases} \tag{3}$$

where $r_i(t)$ represents the reputation of player i at time step t, and $s_i(t)$ and $s_j(t)$ denote the strategies employed by players i and j respectively in the interaction. In this context, the parameter δ assumes a pivotal role as a reputation adjustment factor. It lies within the range $[0, 1]$.

The update mechanism operates based on the outcomes of pairwise interactions within initiated groups. When player i cooperates ($s_i(t) = 1$) while their partner j defects ($s_j(t) = 0$), player i's reputation is augmented by δ. Conversely, if player i defects while player j cooperates, player i's reputation diminishes by δ. However, if both players cooperate or both defect, there is no change in reputation. This can be explained as: in situations where actions are predictable and align with expectations, there is typically no significant impact on reputation. For example, if two business partners consistently uphold their agreements, their reputations for reliability and trustworthiness remain stable. Likewise, if they consistently fail to meet expectations, their reputations for unreliability might also remain unchanged.

Additionally, to maintain the meaningfulness of reputation scores and prevent distortion of the reputation dynamics, a constraint is imposed such that reputations remain bounded within the range of 0 to 2. Any reputation values exceeding 2 are capped at 2, while values falling below 0 are set to 0. This constraint serves to stabilize the reputation system and prevent reputations from becoming excessively inflated or deflated.

For non-initiated (G_0) groups, the reputation update process diverges slightly from that of initiated (G_1) groups. In non-initiated groups, the reputation of defectors experiences a reduction by δ, while cooperators' reputations remain unaltered. Mathematically, this can be expressed as

$$r_i(t+1) = \begin{cases} r_i(t) - \delta, & s_i(t) = 0, \\ r_i(t), & s_i(t) = 1. \end{cases} \tag{4}$$

Consequently, the total reputation for player i is represented as

$$R_i(t+1) = \begin{cases} r_i(t) + \delta \sum_{i\in g\in G_1} (1 - s_i(t)), & s_i(t) = 1, \\ r_i(t) - \delta m_0 - \delta \sum_{i\in g\in G_1} s_i(t), & s_i(t) = 0. \end{cases} \tag{5}$$

In this equation, m_0 represents the number of non-initiated groups, and R_i denotes the total reputation of player i, encompassing both initiated and non-initiated groups. For cooperators ($s_i = 1$), the total reputation is the sum of reputations within initiated groups. For defectors ($s_i = 0$), the total reputation is the sum of reputations within initiated groups, while accounting for the reduction in reputation (δ) incurred in non-initiated groups.

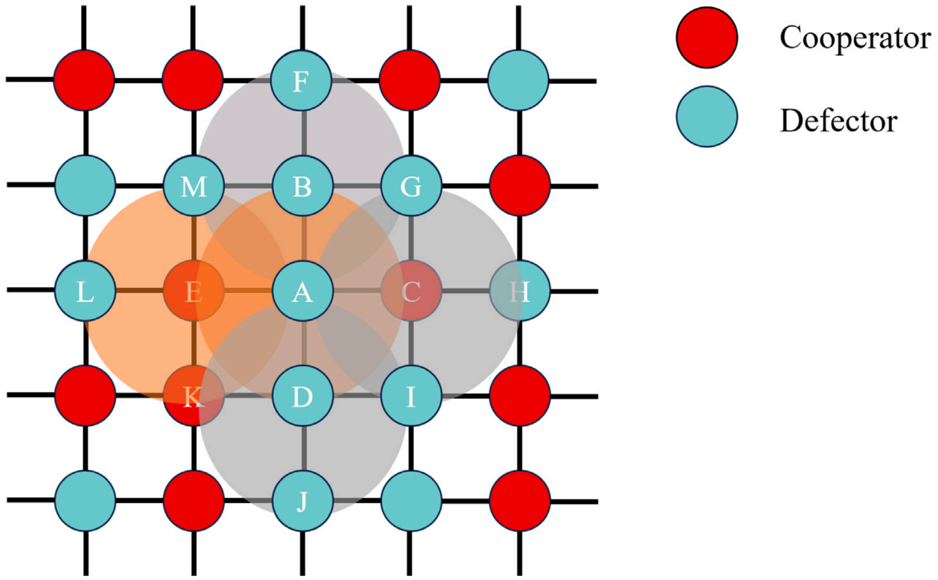

Fig. 1. An example of a 5 × 5 square lattice, populated with cooperators and defectors denoted by red circles (C) and blue circles (D), respectively. The light gray shading indicates groups that have not yet successfully initiated, while the light orange shading represents those that have initiated successfully. The simulation is configured with settings $R = 2$, $\phi = 2$, and $\delta = 0.01$.

2.4. *Strategy learning*

To amalgamate the influences of the threshold and reputation parameters on strategy learning, we introduce the fitness parameter F_i in our model [49–52]. The individual fitness F_i as the product of the player's reputation R_i and the payoff P_i can be computed by

$$F_i = R_i \times P_i. \tag{6}$$

Subsequently, the probability that player i adopts the strategy s_j of a randomly chosen player j within the initiated (G_1) groups is determined by

$$W(s_i \leftarrow s_j) = \frac{1}{1 + e^{-(F_j - F_i)/\kappa}}, \tag{7}$$

where $\kappa \in [0, \infty)$ is a positive constant indicating the uncertainty in strategy adoption. A higher κ introduces more randomness or noise in the decision-making process, making strategy adoption less deterministic and more influenced by factors beyond just the fitness difference.

In our work, we mainly focus on the fraction of cooperators

$$f_C(t) = \frac{N_C(t)}{N}, \quad t = \{0, 1, \ldots, \infty\}, \tag{8}$$

where N_C represents the number of cooperators, and f_C is the level of cooperation. Thus, the value of f_C lies between 0 and 1, with 0 representing the absence of cooperators and 1 indicating an entire population of cooperators. In addition, $\overline{f_C}$ refers to the average level of cooperation in the stable state.

Fig. 1 illustrates the computation of player A's payoff and reputation within the context of the spatial Public Goods Game with parameters $R = 2$, $\phi = 2$ and $\delta = 0.01$. In the lattice, cooperative individuals are denoted by red circles, while defective counterparts are represented by blue symbols. The focal player A involves in five groups, including three non-initiated G_0 groups (i.e., $\{A, M, B, G, F\}$, $\{A, G, C, I, H\}$, and $\{A, K, D, I, J\}$), and two initiated G_1 groups (i.e., $\{A, M, E, K, L\}$ and $\{A, B, C, D, E\}$). In these three G_0 groups, player A receives zero payoffs, and its reputation reduces by $3\delta = 3 \times 0.01 = 0.03$. However, in these two G_1 groups, player A receives payoffs of double $(2 \times 2/5 - 1)$, i.e., -0.4, and its reputation decreases by $4\delta = 4 \times 0.01 = 0.04$.

3. Results

3.1. *Initiation threshold*

At the initial time step $t = 0$, our lattice is initially populated with a balanced mix of cooperators (C) and defectors (D), resulting in a uniformly distributed strategic configuration.

Our investigation reveals intriguing dynamics contingent upon the initiation threshold ϕ. From Fig. 2(a), we can observe that, for a lower threshold (e.g., $\phi = 0$), cooperation levels decline to 0. In the case of a middle threshold (e.g., $\phi = 2$), there is an initial decrease in cooperation levels, and then cooperators start forming compact clusters, and this clustering phenomenon assists cooperators in gradually overpowering defectors. The above phenomenon aligns with the well-acknowledged "Enduring" (END) and "Expanding" (EXP) phases observed in spatial cooperation games. The significance of earlier works in understanding network reciprocity dynamics can be referred to [53,54]. In contrast, for a higher threshold (e.g., $\phi = 4$), a noticeable and consistent upward trend in cooperation levels is observed over time. Remarkably, at the extreme value of $\phi = 5$, the cooperation level remains constant at 0.5.

3.2. *Synergy factor*

The synergy factor R significantly influences the payoff dynamics in each PGG, playing a pivotal role in shaping cooperative behavior. Fig. 3 shows the average level of cooperators $\overline{f_C}$ as a function of the synergy factor R for different values of ϕ. When $\phi = 0$, indicating that the system does not impose any requirements on the minimum contribution or number of cooperators needed for groups within the game, it corresponds to the ordinary case where all groups can be initiated. Notably, there exists a critical synergy point R_c, beyond which cooperation becomes a favorable and sustainable strategy. With the increase of ϕ, a smaller R_c is sufficient to guarantee the emergence of cooperation, eventually leading to the system achieving global cooperation. This positive influence of the initiation threshold mechanism on the emergence of cooperation has been verified in Fig. 2(b).

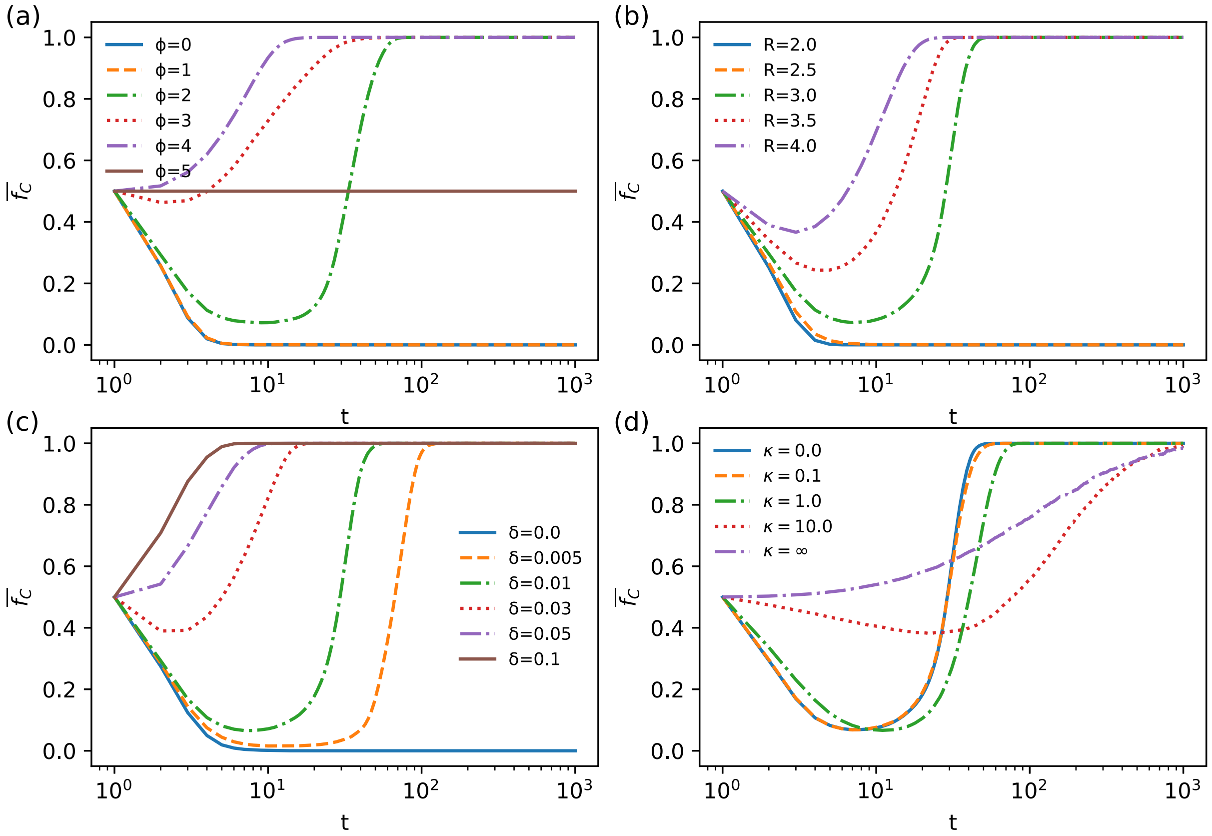

Fig. 2. The average cooperation level $\overline{f_C}$ evolves over time t in a lattice with the total time $T = 1000$. Settings: (a) $R = 2.3$, $\delta = 0.01$, $\kappa = 0.1$; (b) $\phi = 1$, $\delta = 0.01$, $\kappa = 0.1$; (c) $\phi = 1$, $R = 3$, $\kappa = 0.1$; (d) $\phi = 1$, $R = 3$, $\delta = 0.01$. Each data point is the average of 50 independent simulations.

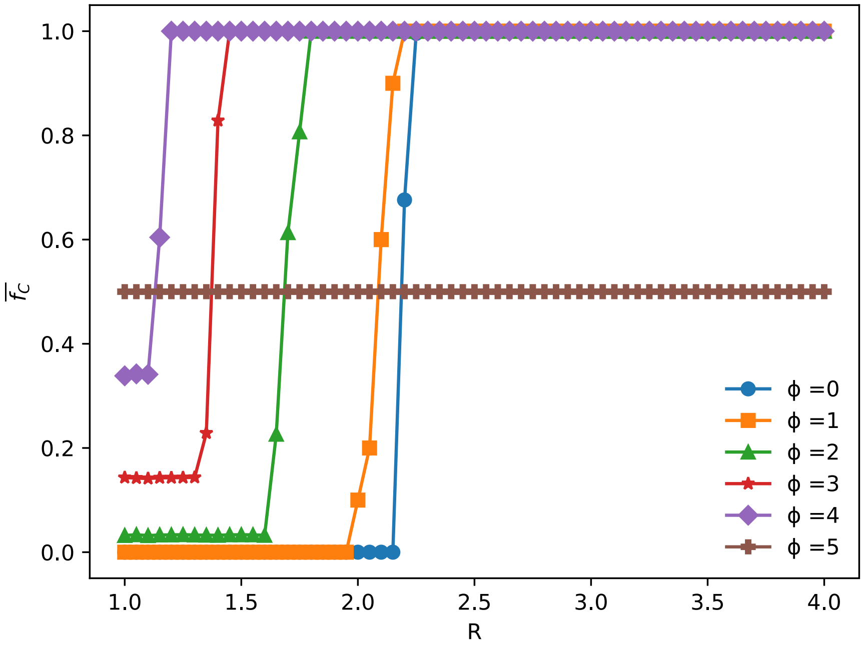

Fig. 3. The average cooperation level $\overline{f_C}$ as a function of the synergy factor R for different values of ϕ. The simulations are conducted on a lattice of size 100×100, with parameters set as $\delta = 0.03$ and $\kappa = 0.1$.

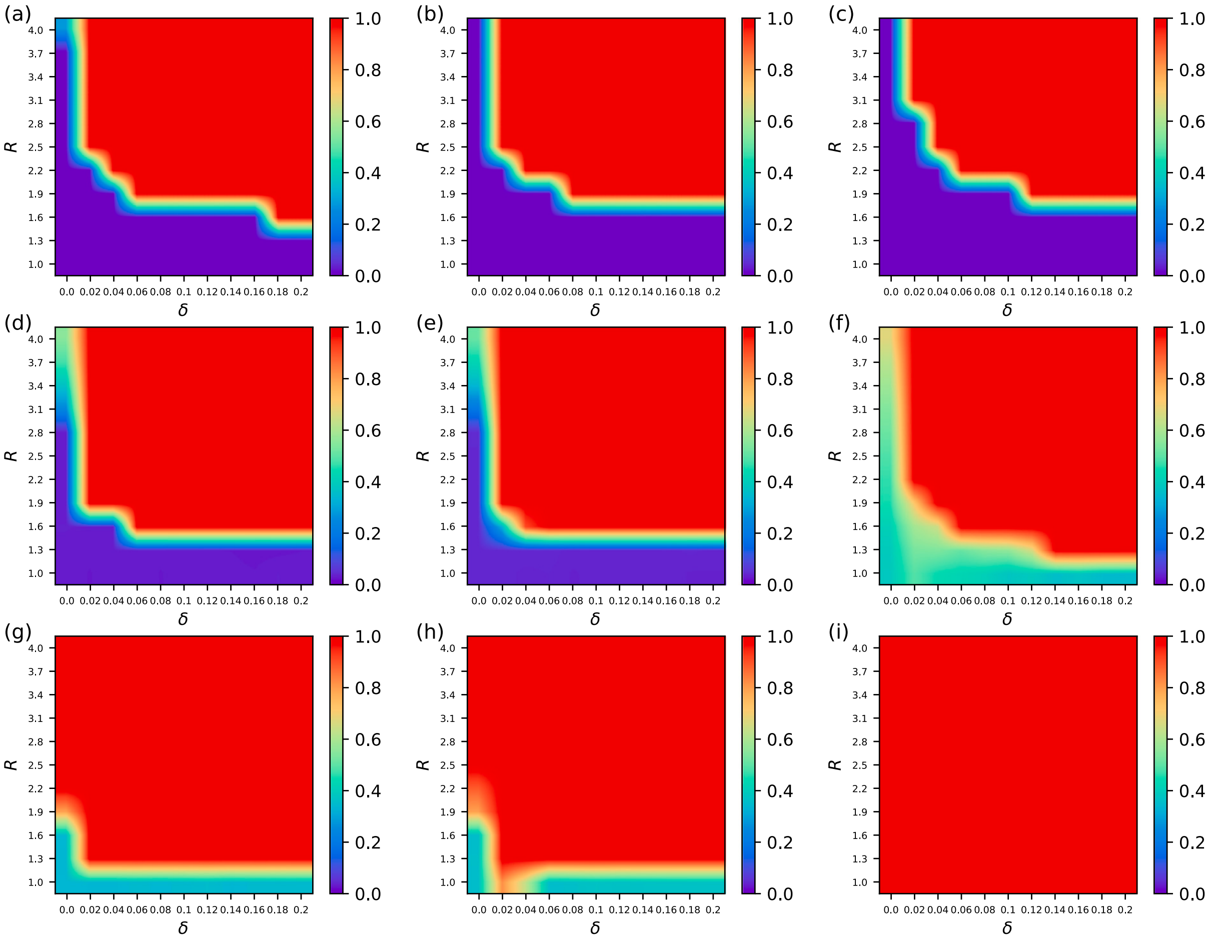

Fig. 4. The heatmap for reputation adjustment factor δ and synergy factor R. The top, middle, and lower panels correspond to $\phi = 0, 2, 4$. Similarly, the panels from left to right showcase different κ values: $\kappa = 0.1$, $\kappa = 1$ and $\kappa = 10$.

3.3. Reputation adjustment factor

The reputation adjustment factor δ plays a crucial role in shaping the dynamics of cooperation within the system.

In scenarios where there is no reputation-based adjustment ($\delta = 0$), the propensity for cooperation diminishes significantly. Without the prospect of reputation changes based on behavior, defectors face no repercussions for their actions, leading to a prevalence of defection strategies within the population. As a result, cooperation struggles to take root, and the system may become entrenched in a state of non-cooperation (as illustrated in Fig. 2(c)).

However, even a small value of δ introduces a shift in the dynamics, facilitating the emergence and stabilization of cooperation (e.g., $\delta = 0.005$ in Fig. 2(c)). This slight adjustment to reputation mechanisms introduces a subtle but meaningful incentive structure. Players now face the prospect of reputation gains or losses based on their cooperative or defective behaviors. Consequently, cooperative individuals are rewarded with enhanced reputations, while defectors incur reputational costs.

The introduction of the adjustment factor, δ, fundamentally alters the landscape of interactions within the system. By diminishing the fitness of defectors, as described in Eq. (6), δ effectively raises the cost of defection. This increase in cost incentivizes individuals to reconsider their strategies, as the benefits of cooperation become more apparent relative to the costs of defection. Over time, this shift in incentives fosters a transition towards cooperative behaviors, leading to the establishment of cooperative norms and the stabilization of cooperation within the population.

3.4. Noise parameter

From the information provided in Fig. 2(d), it is evident that the influence of noise κ on strategy imitation significantly affects the timing of cooperation emergence in the system. Specifically, when the noise κ is small (e.g., $\kappa = 0, 0.1, 1.0$), indicating that imitators can more accurately mimic players with higher individual fitness, the steady state of cooperation often materializes earlier. On the contrary, for larger noise values (e.g., $\kappa = 10$), the steady state of cooperation occurs later or becomes more challenging to reach and stabilize.

3.5. Heatmaps

We engage in an extensive examination of the intricate factors influencing the overall level of cooperation through the utilization of heatmaps. These visual representations are valuable in revealing the complexities of the parameter space and elucidating how diverse values of parameters contribute to cooperative dynamics.

In Fig. 4, we depict a heatmap that intricately illustrates the interplay between the reputation adjustment factor (δ) and the synergy factor (R). This diagram serves as a crucial analytical tool, providing insights into the cooperative dynamics of spatial public goods games under various strategic conditions. The figure is segmented into three parts (top, middle, and lower), each corresponding to different initiation threshold values ($\phi = 0, 2, 4$). This categorization enables a nuanced understanding of how the initiation threshold influences cooperation dynamics. Furthermore, within each part, we observe variations across uncertainty parameter (κ) values ($\kappa = 0.1, 1, 10$) from the left to the right, offering additional insights into the role of uncertainty in strategy adoption on overall level of cooperation dynamics. The combined consideration of δ, R, ϕ, and κ in each panel of Fig. 4 provides a holistic view of the game's equilibrium states.

To delve into the specific findings, the initiation threshold ϕ emerges as a significant determinant of cooperation. Higher values of ϕ are found to amplify cooperation substantially. For instance, when ϕ is sufficiently large (e.g., $\phi = 4$), the region of complete cooperation (where $\overline{f_C} = 1$) spans the entire parameter space, as vividly depicted in Fig. 4(g,h,i).

Additionally, the noise parameter κ assumes a pivotal role in shaping the cooperation level. Counterintuitively, higher uncertainty (larger values of κ) enhances cooperation levels under higher thresholds (e.g., $\phi = 4$). This is because a higher threshold trigger in the initiated groups, a higher probability that an individual would choose the cooperators to learn from. This observation becomes apparent in the heatmaps, where the purple regions (indicating the absence of cooperation) shrink or the red regions (indicating higher levels of cooperation) expand as the value of κ increases.

4. Conclusion

To summarize, our examination of the threshold-based spatial Public Goods Game model has shed light on crucial factors shaping cooperation dynamics within spatially structured environments. The intricacies of interactions, driven by cooperative thresholds, reputation adjustment factors, and strategy learning noise, unveil a diverse landscape of cooperative behavior that surpasses conventional expectations.

The significance of higher initiation thresholds in fostering cooperation suggests the importance of establishing a cooperative contribution baseline within groups. This insight bears practical relevance for scenarios necessitating cooperation, such as community-based initiatives, organizational settings, or ecological conservation efforts. By comprehending the impact of initiation thresholds, policymakers and organizers can devise interventions that effectively promote cooperative behavior.

The introduction of a reputation adjustment factor emerges as a powerful tool for shaping cooperative behaviors. The dual impact on defectors, both within non-initiated PGG and through judgments by cooperators within initiated PGG, underscores the nuanced interplay between reputation and cooperation. This understanding can inform the design of incentive structures or governance mechanisms that leverage reputation to encourage cooperation in diverse settings.

The unexpected role of noise in strategy imitation adds a layer of complexity to our understanding of cooperative dynamics. Higher uncertainty in strategy adoption under higher thresholds leading to increased cooperation challenges conventional wisdom and prompts a reevaluation of the role of randomness and uncertainty in social systems.

Looking forward, refining the model to accommodate varying initiation thresholds for different groups and integrating more sophisticated reputation dynamics could offer a more nuanced understanding of cooperative behaviors. Additionally, extending the application of insights garnered from this study to targeted real-world scenarios, such as sustainable resource management, urban planning, or online collaborative platforms, presents an opportunity to devise tailored strategies that leverage cooperative dynamics effectively.

In essence, our exploration not only contributes to the theoretical framework of spatial public goods game but also offers practical guidance for promoting and sustaining cooperation in a variety of complex, real-world scenarios. The intricate interplay of initiation thresholds, reputation adjustment factors, and strategy learning noise provides a multifaceted perspective on cooperative behavior, laying the groundwork for further advancements in the study of cooperation in spatially structured environments.

CRediT authorship contribution statement

Weijie Wang: Visualization, Validation, Methodology, Conceptualization. **Zhehang Xu:** Visualization, Validation, Software, Conceptualization. **Shijia Hua:** Writing – review & editing, Writing – original draft. **Longqing Cui:** Writing – review & editing, Writing – original draft, Conceptualization. **Jianlin Zhang:** Supervision, Methodology, Conceptualization. **Fanyuan Meng:** Writing – review & editing, Writing – original draft, Supervision, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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