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The coordination of a dual-channel container transportation service chain with an option contract

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ABSTRACT

This paper examines empty container capacity planning and channel coordination in a dual-channel container transportation service chain (DCCTSC) in the context of random market demand. We first develop a benchmark model that does not take into account the option contract. Then, we introduce the option contract between the forwarder and the carrier and establish empty container leasing models in the decentralised and centralised modes. Next, we introduce an improved revenue-sharing contract based on the model constructed above to investigate the coordination of the revenue-sharing option contract in the DCCTSC. We obtain the relational equation for the contract parameters that is satisfied when the system is coordinated and analyse the Pareto improvement interval for the contract parameters that enable the carrier and the forwarder to attain a mutually beneficial outcome. Finally, the conclusion is confirmed by numerical simulation. The outcomes indicate that the option contract is effective in reducing the risk to the forwarder caused by market uncertainty, leading to an approximately 16-fold increase in its profit. The revenue-sharing option contract can reasonably distribute the system's profits, resulting in a win–win outcome for both the carrier and the forwarder, increasing the total system profit by up to 6.45%.

1. Introduction

In order to adapt to the development of the shipping market, container leasing companies, carriers, forwarders and other enterprises to gradually develop cooperation, ultimately forms a comprehensive and efficient multimember service supply chain around container transportation. We call this the container transportation service chain [1]. In the traditional container transportation service chain system, the forwarder acts as an intermediary between the carrier and the consignor, assisting the carrier in canvassing, while the carrier is responsible for providing transportation services to the consignor, which is the traditional canvassing channel. Nowadays, some carriers are also implementing a "dry port" strategy, where they directly engage with consignors for canvassing, bypassing the involvement of forwarders [2]. This represents the carrier's direct canvassing channel. These two methods of canvassing undoubtedly strengthen the carrier's multi-channel approach to cargo acquisition and naturally form a dual-channel container transportation service chain (DCCTSC) [3]. Although the dual-channel approach enables better response to the transportation demands in the shipping market, it also gives rise to certain issues. On the one hand, the carrier is the upstream enterprise of the forwarder, they will make decisions based on their own interests, which may disregard the overall benefits of the container transportation service chain, leading to the occurrence of "double marginal effects". On the other hand, both the carrier and the forwarder have the capability to canvass for cargoes directly from consignors, resulting in horizontal competition between them, which can trigger channel conflicts. Therefore, how to effectively coordinate the intricate relationship between the carrier and the forwarder to leverage the advantages of the two channels and achieve efficient and profitable operations of the DCCTSC is a pressing issue that requires immediate attention and resolution.

Carriers and forwarders typically determine the disposable quantity of empty containers before demand arises. However, the uncertainty in shipping demand makes this decision regarding empty containers challenging, and carriers and forwarders are prone to situations where insufficient empty containers lead to revenue loss or excess inventory results in wastage of container resources. This negative impact is particularly pronounced in a complex environment such as dual-channel. Consequently, it is crucial for us to explore how members within the DCCTSC can make reasonable quantity planning

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of empty containers amid demand uncertainty.

Due to the fleeting nature of transportation demand, empty containers do not have a residual value, and redundant empty containers will incur storage costs, the need for flexibility in empty container leasing contracts is more urgent, so we introduce option contracts into the DCCTSC. In the traditional channel, the forwarder orders a certain quantity of containers from the carrier at a pre-negotiated option order price, and then executes the contract at a negotiated option exercise price after demand is confirmed. For the carrier, this process brings some benefits. First, the forwarder orders a certain number of options in advance, which serves as a valuable reference for the carrier in determining appropriate container leasing quantities. This approach mitigates the forwarder's risks associated with excessive or insufficient leasing of empty containers to some extent. Second, the option order fee paid in advance by the forwarder provides partial funds for the carrier to lease containers, alleviating the capital pressure on the carrier. Option contracts also have two distinct advantages for the forwarder. First, the forwarder can adjust the number of options to be exercised after the sales season begins according to market demand, which mitigates the risk associated with market demand uncertainty. Second, the forwarder's prior payment of the option ordering fee ensures the accessibility of empty containers. The predetermined option exercise price further avoids the risk of market price fluctuations. Therefore, in the context of uncertain demand, it is evident that designing a scientific, rational, and effective contract through the consideration of option contracts to achieve coordination in the DCCTSC is a significant research topic worthy of attention.

In summary, building upon the study conducted by Luo et al. [3], this research extends the investigation to consider the impact of option contracts on empty container planning and channel coordination within the DCCTSC. First, we separately analyse the optimal key decisions of each member in the dual-channel system before and after the application of option contracts, identifying crucial relationships between the important parameters and the decision variables. Subsequently, we explore the optimal level of empty containers for the carrier and the forwarder in the centralised mode and use this as the highest standard after coordination. Continuing, we effectively amalgamate option contract with revenue-sharing contract, forming an efficient combined contract that achieves perfect coordination within the dual-channel system, and identify the conditions for successful coordination. Finally, we analyse the Pareto improvement intervals within which this contract can be implemented.

The contributions of this research are summarised as follows:

(1) We apply the option contract, which is more maturely applied in the traditional product supply chain, to the maritime service chain, specifically between the traditional channel members of the DCCTSC, in order to explore the risk-sharing mechanism of the option contract between the forwarder and the carrier. It not only provides new insights for the forwarder to order option capacity, manage empty containers and improve competitiveness, but also expands the potential uses of option contracts.
(2) We investigate the DCCTSC by additionally considering the direct canvassing channel on the basis of the traditional canvassing channel. After comprehensively considering the co-existence of competition and cooperation between the carrier and the forwarder due to the addition of direct channel, we propose an optimal empty container planning strategy for the whole dual-channel service chain system. This provides a novel approach for the rational planning of empty container resources for members of the container transportation service market.
(3) We integrate option contract and revenue-sharing contract, employing a contract coordination approach to address the empty container resource planning issue in the DCCTSC. By setting appropriate values of contract parameters, we enable both the carrier and the forwarder share the risk and revenue, thereby attaining the comprehensive optimization of container resources. This represents an external, decentralised decision-making research perspective, distinct from the internal resource combination optimization mindset of a single central planner under centralised decision-making. This research approach is closer to the reality, better integrates the container resources in the shipping market, and also enlarges the domain of applications for contract coordination theory.

2. Literature review

The first group of literature closely associated with this study is the research related to option contracts. Option contracts give the purchaser the right to buy or sell goods at an agreed price at a specific time. As it enhances the ability of supply chain members to cope with market uncertainty and better coordinates the interests of supply chain members, it has attracted extensive attention from scholars. Currently, research on option contracts focuses on traditional product supply chains. Depending on the different forms of the option, option contracts in product supply chains can be separated into the call option contract, the put option contract and the bidirectional option contract.

Among the studies on call option contracts, Barnes-Schuster et al. [4] were the first scholars to probe the harmonization of supply chains through option contracts. Their study showed that option contracts can provide sellers with good flexibility to respond to market demand uncertainty and improve supply chain flexibility and achieve supply chains harmonization. Chen et al. [5] investigated how option contract can effectively harmonise a secondary supply chain under the condition of risk aversion by retailers. They found that the optimal order volume of retailers under risk aversion was lower than when under risk neutrality. Luo et al. [6] explored the coordination of option contract in a seller-led supply chain under the spot market. They focused on the impact of spot market supply risks and price risks on the decision of both suppliers and sellers. In the study of put option contracts, Eppen and Iyer [7] were the first to borrow the idea of a put option and devise a new type of contract, the back-up contract. In a back-up contract, if the seller observes that demand at the beginning of the season is not as optimistic as expected, it can cancel a portion of the order by paying a default fee to the supplier. Hu et al. [8] introduced put option contracts into a supply chain system made up of government and emergency supply storage companies and demonstrated that put option contracts can coordinate the supply chains of disaster relief supplies. On this basis, they also compared the mechanisms of put option contracts, wholesale price contracts and repurchase contracts, illustrating that put option contracts are more flexible relative to other contracts and can effectively reduce the risk of emergency material inventory. In the study of bidirectional option contracts, Wang et al. [9] discussed the impact of bidirectional option contracts on newspaper return prices and customer ordering decisions, deriving the variation of optimal return prices and optimal order quantities with option prices, strike prices and wholesale prices. The findings also show that bidirectional option contracts can reduce the negative impact of customer earnings and improve profits.

Although the above studies involve various forms of option contracts, they all consider only the impact of the option contract itself on the supply chain, ignoring the possibility that a single option contract may not mitigate the effects of the double marginal effect. As a result, some researchers have turned their attention to the issue of combining option contracts with other contracts. Zhang and Hua [10] compared the financing strategies of financially constrained retailers and supply chain coordination strategies under individual and combined contracts, showing that individual contracts cannot coordinate supply chains while combined contracts can. Arani et al. [11] introduce a hybrid mechanism of a revenue-sharing contract and an option contract, incorporating immediate purchase and shortage penalty mechanisms, drawing the optimal decisions and coordination conditions under different member dominance, and comparing them with two separate contracts, finding that the hybrid contract outperforms the other two. However, while these two studies explore combination contracts that include option contracts, have both focused on single-channel structures. The research on the impact of combination contracts with option contracts in a multi-channel environment is virtually nonexistent.

In addition, only a few scholars have introduced option contracts into studies related to maritime transportation chains, but their emphasis remains on the mechanism of a single option contract in the maritime transportation chain. For example, Liu et al. [12] designed a two-stage option contract mechanism from the carrier's perspective based on the characteristics of the shipping market and proposed corresponding option purchase recommendations for the peak and low shipping seasons. While Luo et al. [13] combine option contract with two-part tariff contract to solve the empty container inventory problem in the shipping service chain and consider option trading as a factor, the structure of the maritime service chain they study is also single-channel.

As indicated in Table 1, we contrast this study with the relevant literature pertaining to option contracts. The above study explores pricing strategies, ordering strategies and coordination issues in options contracts, covering call options, put options and bidirectional options, and includes hybrid markets consisting of option contract markets and spot markets, and hybrid contracts are made up of option contracts and other basic contracts. However, most research has focused on single-channel supply chains, and there is a dearth of research on option contracts in the dual-channel arena. Furthermore, the literature applying option contracts to maritime transportation chains is limited. Either they do not consider joint contracts or dual-channel forms of canvassing in maritime chains. Therefore, we bring option contracts into the DCCTSC to explore its ordering and coordination issues, which is a great supplement to the study of option contracts and adds more practical significance.

The second category of literature closely related to this study pertains to research on resource optimisation in maritime supply chains. Currently, the existing literature in this domain predominantly addresses the internal resource optimisation problems of individual decision-makers within maritime supply chains. In essence, this represents a centralised mode of internal optimisation decision-making. For instance, Zheng et al. [14] proposed a two-stage optimisation approach, where the first stage involves centralised allocation of empty containers, followed by the determination of the cost of transporting empty containers. This approach aims to coordinate the empty container resources among multiple shipping companies, thereby reducing the likelihood of empty container repositioning. Trapp et al [15] investigated the impacts of cooperation and competition among members of the maritime supply chain on the economy and environment, respectively, by constructing an integer optimisation model based on a combination of both economic and environmental factors. Chen et al. [16] discussed alliances between carriers from the perspective of slot sharing and based on this, a column generation algorithm was used to solve the problem of capacity scheduling between alliances. Wang et al. [17] took a port-centred shipping supply chain as the research object and proposed an integrated scheduling scheme based on an improved genetic algorithm to solve the multimodal transport scheduling problem when chain members form alliances.

Another small portion of research around resource optimisation can be summarised as collaborative management studies in the maritime supply chain. Common collaboration modes include horizontal collaboration between carriers and carriers, forwarders and forwarders, and ports and ports, as well as vertical collaboration between carriers and ports, forwarders and carriers. They typically employ game theory and contract coordination theory to achieve collaboration, representing a decentralised external decision-making approach. Among these, research on horizontal collaboration is more prevalent. Song et al. [18] built a static cost model between two independent ports and a competitive game cost model for the transport supply chain made up of two ports and one carrier, which provides an effective reference for port pricing strategies and supply chain integration policies. Liu and Wang [19] studied the horizontal alliance value between carriers in a highly competitive shipping industry. They utilised a hybrid contract to harmonise ocean freight chains and judged the effect of this contract on the horizontal alliances of carriers. Wang and Liu [20] investigated how to make appropriate contractual choice decisions to maximise the benefits of two shipping supply chains when horizontal competition between carriers evolves into service competition between two parallel shipping supply chains. In terms of vertical collaboration, Liu et al. [12] took a single-cycle shipping service chain consisting of a leasing company and a carrier, and increased the transaction volume between the carrier and the leasing company by designing a leasing contract. This is actually a vertical collaboration between the carrier and the upstream leasing company.

As outlined in Table 2, our study is contrasted with the literature concerning resource optimisation in maritime supply chains. It is evident that most of the research on resource optimisation in the maritime supply chain is based on the idea of maximising or minimising single-subject objectives in operations research, supplemented by various intelligent algorithm designs or simulation techniques to seek optimal or near-optimal solutions for various resources within individual entities. However, research on the optimal allocation and rational planning of overall resources in maritime supply chains through decentralised external decision-making, coupled with game theory and contract design, is relatively scarce. In addition, existing research predominantly focuses on single-channel maritime supply chains, with limited exploration into the realm of dual-channel maritime supply chains. In contrast, our study introduces the direct canvassing channel of carriers, delving into the coordination between the two channels and the collaboration between carriers and forwarders to create optimal empty

Table 1
A comparison between this study and literature on option contracts.

Reference	Research object		Channel structure		Contract type	
	Traditional product supply chain	Maritime transportation chain	Single-channel	Dual-channel	Separate option contract	Option contract & other contracts
Barnes-Schuster et al. [4]	✓		✓		✓	
Chen et al. [5]	✓		✓		✓	
Luo et al. [6]	✓		✓		✓	
Eppen and Iyer [7]	✓		✓		✓	
Hu et al. [8]	✓		✓		✓	
Wang et al. [9]	✓		✓		✓	
Zhang and Hua [10]	✓		✓			✓
Arani et al. [11]	✓		✓			✓
Liu et al. [12]		✓	✓		✓	
Luo et al. [13]		✓	✓			✓
This study		✓		✓		✓

Table 2
A comparison between this study and literature on resource optimisation in maritime supply chains.

Reference	Solution method	Decision-making method		Channel structure	
		Internal decision-making	External decision-making	Single-channel	Dual-channel
Zheng et al. [14]	Two-stage optimisation method	✓		✓	
Trapp et al. [15]	Integer optimisation model	✓		✓	
Chen et al. [16]	Column generation algorithm	✓		✓	
Wang et al. [17]	Multiple heterogeneous coding genetic algorithm	✓		✓	
Song et al. [18]	Game model		✓	✓	
Liu and Wang [19]	Revenue sharing & service cost allocation contract		✓	✓	
Wang and Liu [20]	Game model		✓	✓	
Liu et al. [12]	Option contract		✓	✓	
This study	Improved revenue-sharing option contract		✓		✓

container plans throughout the entire system.

The third category of literature closely related to this study pertains to research on supply chain contracts. Coordination is the key question of supply chain management, and contracts are effective mechanisms to accomplish supply chain coordination. Currently, there is a substantial body of literature utilising contract coordination in traditional product supply chains to eliminate double marginalisation effects or alleviate the bullwhip effect. This literature not only addresses single-channel supply chains but also extends to dual-channel supply chains, employing a diverse array of contract types. For instance, Palsule-Desai [21], Ma et al. [22], and Zheng et al. [23] addressed coordination issues in single-channel supply chains using a revenue-sharing contract, a two-part tariff contract, and a self-designed contract capable of sharing capacity and demand information, respectively. Sarathi et al. [24] coordinated a single-channel two-stage short lifecycle product supply chain using a joint contract model that integrates a revenue-sharing contract and a quantity discount contract. Chen et al. [25] demonstrated how a single contract with supplementary agreements, such as a profit-sharing contract, could effectively coordinate a dual-channel supply chain. Also, Feng et al. [26] and Zhu et al. [27] employed joint contracts to address coordination issues in dual-channel supply chains under different scenarios. However, there is limited literature employing supply chain contracts as a tool to address resource coordination issues in shipping chains. Only a few studies, such as that by Xie et al. [2], have explored this avenue. However, their focus is on using a bilateral buy-back contract to address the problem of sharing empty container inventory between railway company and liner company in the rail-sea intermodal transportation. They neither considered dual-channel canvassing scenarios nor attempted the use of combined contracts.

As indicated in Table 3, we contrast this study with the relevant literature pertaining to supply chain contracts. It is evident that research on contract coordination in traditional product supply chains is well-established. Academics have extensively extended fundamental contract models for different supply chain scenarios, proposing and developing various derivative, enhanced, and composite contract models. The ultimate goal is to enhance the efficiency and profitability of the supply chain. However, there is a paucity of studies on contract coordination specifically in the maritime domain. The limited existing studies in this area have also not delved into the context of dual-channel maritime transport. In accordance with this, we devised a combined contract that incorporates the characteristics of dual-channel maritime chains and integrates option contracts. This contract is designed to address the coordination and planning of empty container resources in the dual-channel canvassing scenario, thereby expanding the usage scenarios of contract coordination theory.

3. Problem description

This paper considers a DCCTSC system comprising a single carrier and a single forwarder. With the exception of the carrier and forwarder, the two main players who need to make decisions, the system is also attended by the container leasing company and downstream customers. The carrier occupies the dominant position in the DCCTSC, which canvasses for cargoes through a combination of direct and traditional channels. The operation process of the direct channel is as follows: the carrier leases empty containers from the container leasing company and stacks them in the yard of the port. Then, the carrier actively implements "dry port" strategy to expand its direct economic hinterland and provide convenient container transportation services for consignors in the inland hinterland. The traditional channel operation process entails the forwarder leasing empty containers from the container leasing company through the carrier and then the forwarder directly to the market customers to complete the canvassing. The joint development of these two channels strengthens the development trend of the carrier in the form of multichannel canvassing, that is, the formation of a DCCTSC, as shown in Fig. 1.

For ease of description, the symbols for the parameters involved in the model in this paper are defined as follows, the subscript f represents the forwarder and the subscript t represents the carrier.

In Table 4, it is essential for the parameters to satisfy the conditions $0 < c_f < w < p_f$, $0 < c_f < v_f + w_f < p_f$, and $0 < c_t < p_t$ to ensure the profitability for both the forwarder and the carrier, i.e., to ensure the feasibility of transactions in both channels. Additionally, akin to Arani

Table 3
A comparison between this study and literature on supply chain contracts.

Reference	Application scenario		Channel structure		Contract style	
	Product supply chain	Shipping chain	Single-channel	Dual-channel	Single contract	Combined contract
Palsule-Desai [21]	✓		✓		✓	
Ma et al. [22]	✓		✓		✓	
Zheng et al. [23]	✓		✓		✓	
Sarathi et al. [24]	✓		✓			✓
Chen et al. [25]	✓			✓	✓	
Feng et al. [26]	✓			✓		✓
Zhu et al. [27]	✓			✓		✓
Xie et al. [2]		✓	✓		✓	
This study		✓		✓		✓

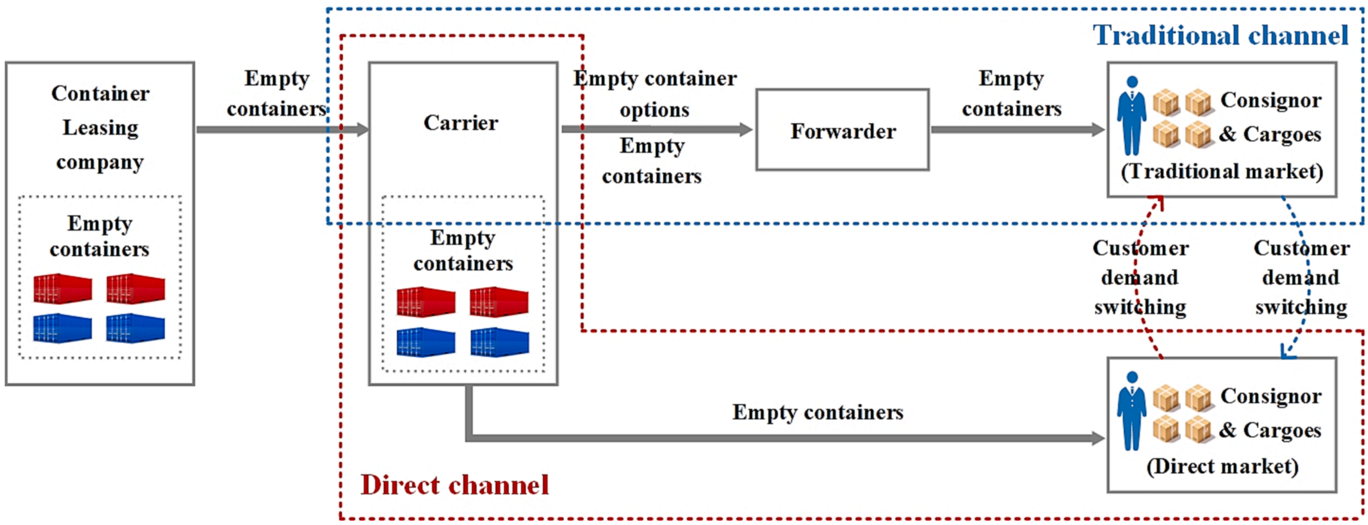

Fig. 1. Flowchart of the DCCTSC considering option contract.

Table 4
Notations.

Variables:	
D_f^*	The market demand of the forwarder in the traditional channel
D_t^*	The market demand of the carrier in the direct channel
D_f	The actual market demand of the forwarder in the traditional channel considering inter-channel demand switching
D_t	The actual market demand of the carrier in the direct channel considering inter-channel demand switching
q_f^0	The quantity of empty container leasing by the forwarder in the absence of considering option contract
q_t^0	The quantity of empty container leasing by the carrier in the absence of considering option contract
q_f	The quantity of empty container options ordered by the forwarder in the traditional channel when considering option contract
q_t	The quantity of empty container options ordered by the carrier in the direct channel when considering option contract
Parameters:	
p_f	Unit cargo canvassing price for container transportation services provided by the forwarder to consignors in the traditional channel
p_t	Unit cargo canvassing price for container transportation services provided by the carrier to consignors in the direct channel
w	Unit wholesale price of empty containers for the forwarder when option contract is not considered
v_f	The unit order price of empty container options for the forwarder
w_f	The unit exercise price of the empty container options for the forwarder
c_f	The unit cost of leasing empty containers for the carrier in the traditional channel
c_t	The unit cost of leasing empty containers for the carrier in the direct channel
g_f	The unit penalty cost when the forwarder fails to meet demand in the traditional channel
g_t	The unit penalty cost when the carrier fails to meet demand in the direct channel
h_f	The unit holding cost of empty containers for the forwarder
h_t	The unit holding cost of empty containers for the carrier
λ_f	The customer transfer rate from traditional channel to direct channel
λ_t	The customer transfer rate from direct channel to traditional channel

et al. [11], we assume $v_f + w_f < w$ to ensure that the forwarder prefers to use option contracts rather than the wholesale price mechanism. The members within the traditional and direct channels are confronted with independent stochastic market demands, denoted as D_f^* and D_t^* respectively. The probability density functions and cumulative distribution functions for the demand in the traditional channel are denoted as $f(x)$ and $F(x)$, respectively. Correspondingly, for the direct channel, these functions are designated as $g(y)$ and $G(y)$. Furthermore, it is assumed that these distribution functions are continuous, differentiable, and strictly increasing.

In the DCCTSC, there is both upstream and downstream cooperation between the carrier and the forwarder, and a horizontal side-by-side relationship. A horizontal competitive relationship means channel competition, so there is some substitution in the container transportation services offered by different channels. Customers will be transferred between the two channels due to the channel's specific supply and demand situation. We denote the customer switching rate by λ_f $(0\leqslant\lambda_f\leqslant1)$ and λ_t $(0\leqslant\lambda_t\leqslant1)$, respectively. Among these, λ_f represents the proportion of transfers from the traditional channel to the direct channel, while λ_t signifies the proportion of transfers from the direct channel to the traditional channel. Similar to the assumptions made by Ernst and Kouvelis [28], Boyaei [29], and Geng and Mallik [30], in order to ensure that it is more profitable for the forwarder and the carrier to prioritise meeting the demands of their own channels, the conditions $\left(p_f + g_f + h_f\right) \geqslant \lambda_f (p_t + g_t + h_t)$ and $(p_t + g_t + h_t) \geqslant \lambda_t \left(p_f + g_f + h_f\right)$ need to be satisfied. After accounting for the demand transfer factors between channels, the actual demand functions for the two channels can be indicated as $D_f = D_f^* + \lambda_t (D_t^* - q_t)^+$ and $D_t = D_t^* + \lambda_f \left(D_f^* - q_f\right)^+$.

4. Decentralised model

In this chapter, we investigate the empty container decisions of each member within the DCCTSC under the decentralised mode. The decentralised mode can also be referred to as the competitive mode. Fig. 2 illustrates the various demand scenarios that arise within each channel when customer demand transfer is taken into consideration.

We focus on analysing the actual satisfied demands, the surplus empty containers, and the unsatisfied demands for both the forwarder and the carrier under different demand circumstances. These three dimensions are closely related to the profit functions of the two members. Specifically, the actual demands satisfied by the forwarder and the carrier can be mathematically represented as $E\min\left\{q_f, D_f\right\}$ and $E\min\{q_t, D_t\}$, respectively. the surplus empty containers left after canvassing by the forwarder and the carrier can be mathematically represented as $E\left(q_f - D_f\right)^+$ and $E(q_t - D_t)^+$, respectively. the unsatisfied demands for the forwarder and the carrier can be mathematically represented as $E\left(D_f - q_f\right)^+$ and $E(D_t - q_t)^+$, respectively.

For the six demand circumstances displayed in Fig. 2, the values of critical parameters are demonstrated in Table 5. We exemplify scenario

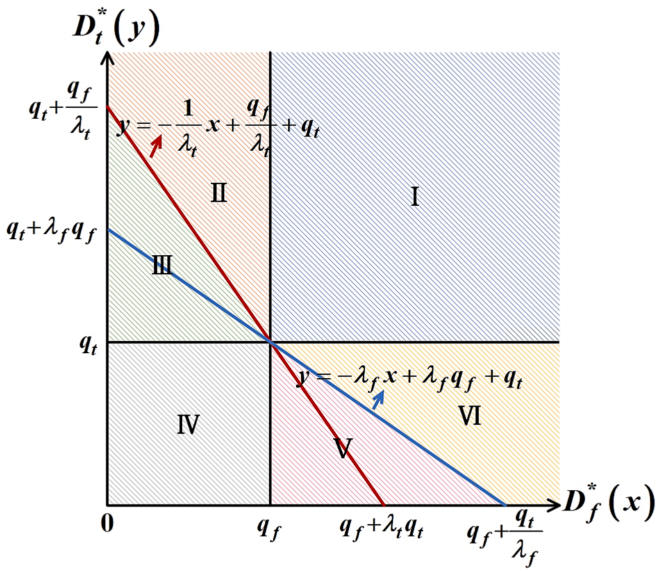

Fig. 2. The demand distribution scenarios faced by each member when considering customer demand transfer.

Table 5
The actual satisfied demands, the surplus empty containers, and the unsatisfied demands under various demand scenarios.

	Scenario I	Scenario II	Scenario III
$E\min\{q_f, D_f\}$	q_f	q_f	$x + \lambda_t(y - q_t)$
$E\min\{q_t, D_t\}$	q_t	q_t	q_t
$E(q_f - D_f)^+$	0	0	$q_f - x - \lambda_t(y - q_t)$
$E(q_t - D_t)^+$	0	0	0
$E(D_f - q_f)^+$	$x + \lambda_t(y - q_t) - q_f$	$x + \lambda_t(y - q_t) - q_f$	0
$E(D_t - q_t)^+$	$y + \lambda_f(x - q_f) - q_t$	$y - q_t$	$y - q_t$

	Scenario IV	Scenario V	Scenario VI
$E\min\{q_f, D_f\}$	x	q_f	q_f
$E\min\{q_t, D_t\}$	y	$y + \lambda_f(x - q_f)$	q_t
$E(q_f - D_f)^+$	$q_f - x$	0	0
$E(q_t - D_t)^+$	$q_t - y$	$q_t - y - \lambda_f(x - q_f)$	0
$E(D_f - q_f)^+$	0	$x - q_f$	$x - q_f$
$E(D_t - q_t)^+$	0	0	$y + \lambda_f(x - q_f) - q_t$

III to elucidate the derivation of these values. In scenario III, it is evident that the demand of the forwarder is less than its inventory while the demand of the carrier exceeds its inventory, i.e., $D_f^* < q_f$ and $D_t^* > q_t$. Since scenario III also satisfies $y < -\frac{1}{\lambda_t}x + \frac{q_f}{\lambda_t} + q_t$, it is equivalent to $q_f - x > \lambda_t(y - q_t)$, meaning that the remaining empty containers for the forwarder are sufficient to fulfil the demand transferred from the direct channel to the traditional channel. Therefore, at this point, the actual demand satisfied by the forwarder is $x + \lambda_t(y - q_t)$. Consequently, it can be deduced that the remaining quantity of empty containers for the forwarder is $q_f - x - \lambda_t(y - q_t)$, and the unfulfilled demand by the forwarder is 0. From the perspective of the carrier, its available quantity of empty containers is inherently unable to fulfil the demand of the direct channel. Under such circumstances, the carrier can at most meet the demand equivalent to its inventory level, denoted as $E\min\{q_t, D_t\} = q_t$. Additionally, no surplus empty containers are available, meaning $E(q_t - D_t)^+ = 0$. Hence, the unfulfilled demand for the carrier amounts to $E(D_t - q_t)^+ = y - q_t$.

4.1. The benchmark model without option contract

To highlight the influence of option contracts on the DCCTSC, we first establish a model that does not consider the option contract, i.e., a traditional wholesale price contract benchmark model. The decision-making process at this point is as follows. First, the carrier forecasts the market demand in the direct channel and decides on the volume of empty containers to be rented in the direct channel. The forwarder then has to make a one-off decision to conclude a container leasing contract directly with the carrier after forecasting the demand in the traditional channel market. The carrier leases empty containers from the container rental company after pooling the total demand from both channels. The sequence of decision-making for the forwarder and the carrier when there is no option contract is shown in Fig. 3.

In the traditional wholesale price contract benchmark model, the forwarder's revenue is derived from the actual sales price in the traditional channel. Its costs mainly include the cost of leasing empty containers from the carrier, the cost caused by being out-of-stock and the cost of holding the surplus empty containers. The carrier's revenue comes from the actual sales proceeds in the direct channel and the revenue from leasing containers to the forwarder. The costs are mainly out-of-stock costs, the storage cost of remaining inventory and the total cost of leasing containers in both channels. The profit function for the forwarder and carrier can therefore be expressed as:

$$\Pi_f^0\left(q_f^0, q_t^0\right) = p_f E\min\left(q_f^0, D_f\right) - wq_f^0 - g_f E\left(D_f - q_f^0\right)^+ - h_f E\left(q_f^0 - D_f\right)^+ \tag{1}$$

$$\begin{aligned}\Pi_t^0\left(q_f^0, q_t^0\right) = {} & p_t E\min\left(q_t^0, D_t\right) + wq_f^0 - g_t E\left(D_t - q_t^0\right)^+ - h_t E\left(q_t^0 - D_t\right)^+ \\ & - c_f q_f^0 - c_t q_t^0\end{aligned} \tag{2}$$

Proposition 1. *In the absence of considering option contract, a scenario arises where $q_t^{0*} = \underset{q_t^0 \in S0}{\arg\max}\, \Pi_t^0(q_t^0)$, leading to the optimal empty container leasing quantities for the forwarder and the carrier being $\left(q_f^{0*}, q_t^{0*}\right)$.*

Proof. Utilising Leibniz's law, we can derive the first and second derivatives of the profit function pertaining to the forwarder as follows:

$$\begin{aligned}\frac{\partial \Pi_f^0\left(q_f^0, q_t^0\right)}{\partial q_f^0} = {} & -h_f \int_0^{q_f^0} \int_0^{-\frac{1}{\lambda_t}x + \frac{q_f^0}{\lambda_t} + q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x - w \\ & + (p_f + g_f)\left[\int_0^{q_f^0} \int_{-\frac{1}{\lambda_t}x + \frac{q_f^0}{\lambda_t} + q_t^0}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^0}^{\infty} \int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right]\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial^2 \Pi_f^0\left(q_f^0, q_t^0\right)}{\partial q_f^{02}} = {} & -(p_f + g_f + h_f) \\ & \cdot \left[\int_0^{q_t^0} f\left(q_f^0\right) g(y)\mathrm{d}y + \frac{1}{\lambda_t}\int_0^{q_f^0} f(x) g\left(-\frac{1}{\lambda_t}x + \frac{q_f^0}{\lambda_t} + q_t^0\right)\mathrm{d}x\right]\end{aligned} \tag{4}$$

It is evident that $\frac{\partial^2 \Pi_f^0(q_f, q_t)}{\partial q_f^2} \leqslant 0$; thus, letting $\frac{\partial \Pi_f^0\left(q_f^0, q_t^0\right)}{\partial q_f^0} = 0$, we can acquire the optimum number of empty containers rented q_f^{0*} by the forwarder without considering the option contract. Next, finding the derivative of $\Pi_t^0\left(q_f^0, q_t^0\right)$ with respect to q_t^0 yields:

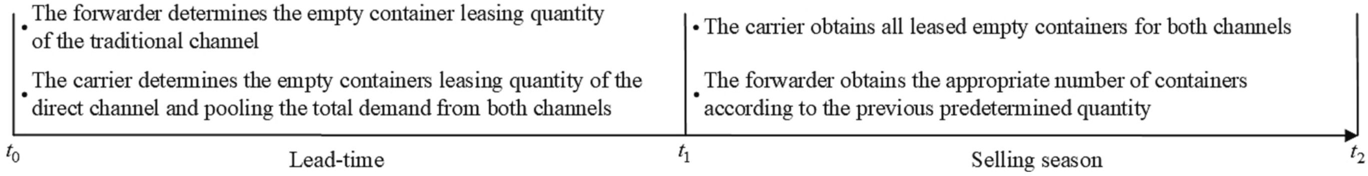

Fig. 3. The decision-making process of the forwarder and the carrier in the absence of option contract.

$$\frac{d\Pi_t^0\left(q_f^0,q_t^0\right)}{dq_t^0}=\frac{\partial\Pi_t^0\left(q_f^0,q_t^0\right)}{\partial q_f^0}\frac{dq_f^0}{dq_t^0}+\frac{\partial\Pi_t^0\left(q_f^0,q_t^0\right)}{\partial q_t^0} \tag{5}$$

From the equation above, it is evident that $\left.\frac{d\Pi_t^0\left(q_f^0,q_t^0\right)}{dq_t^0}\right|_{q_f^0=0}=p_t+g_t-c_t>0$ and $\left.\frac{d\Pi_t^0\left(q_f^0,q_t^0\right)}{dq_t^0}\right|_{q_f^0=\infty}=-h_t-c_t<0$. Furthermore, due to the continuity of the function $\frac{d\Pi_t^0\left(q_f^0,q_t^0\right)}{dq_t^0}$, there is at least one solution that results in $\frac{d\Pi_t^0\left(q_f^0,q_t^0\right)}{dq_t^0}=0$. Hence, there exists a value $q_t^{0^*}=\underset{q_t^0\in S0}{\arg}\max\Pi_t^0\left(q_t^0\right)$ such that under a traditional wholesale price contract, there exists an optimal set of empty container leasing quantities for both the forwarder and the carrier, denoted as $\left(q_f^{0^*},q_t^{0^*}\right)$. Here, $S0$ represents the set of q_t^0 that satisfies the condition $\frac{d\Pi_t^0\left(q_f^0,q_t^0\right)}{dq_t^0}=0$. Q.E.D.

4.2. The DCCTSC model considering option contract

When the option contract is introduced, the carrier's decision in the direct channel will not change. In contrast, the decision of the forwarder in the traditional channel will change from a one-off decision to a two-stage decision. First, at the beginning of the lead time t_0, the carrier decides the empty container leasing volume q_t for the direct channel in accordance with the forecast of the market demand. The forwarder determines the empty container option ordering volume q_f for the traditional channel in light of the forecast of the market demand. The forwarder signs a container ordering contract with the carrier, i.e., the empty container option contract. In this case, the forwarder is required to pay the carrier empty container option purchase fee to obtain the right to get an appropriate volume of empty containers in the future. The carrier then pays the container leasing company the leasing fee for the total estimated empty containers required for both channels and then obtains q_f+q_t empty containers that are stored in the port's yard. At the beginning of the selling season t_1, the forwarder determines the number of empty container options that will be exercised based on the actual demand in the market, making an adjustment to the size of the empty containers lease that does not surpass the number of options purchased. To obtain empty containers, the forwarder needs to pay the carrier a preagreed price for the exercising of the option. The carrier then delivers the exercised volume of empty containers to the forwarder. The sequence of decisions for the forwarder and carrier when considering option contracts is shown in Fig. 4.

In the decentralised mode considering the option contract, the earnings of the forwarder are mainly derived from the actual sales price of empty containers in the traditional channel, and the costs mainly include the cost of ordering and executing empty container options, as well as the cost of out-of-stock in the traditional channel. The carrier's revenue mainly arises from the actual sales price of empty containers in the direct channel, the proceeds from the sale of options to the forwarder, and the benefits from empty container options being exercised. Its costs mainly include empty container leasing cost, the cost of shortage of containers in the direct channel and the holding cost of surplus empty containers. Among them, as the forwarder orders empty container options from the carrier rather than empty containers and the empty containers that the forwarder has not exercised will always be stacked in the port's yard, the storage cost of remaining empty containers in the traditional channel is borne by the carrier. At this point, the profit functions of the forwarder and the carrier can be expressed separately as:

$$\Pi_f\left(q_f,q_t\right)=\left(p_f-w_f\right)E\min\left(q_f,D_f\right)-v_fq_f-g_fE\left(D_f-q_f\right)^+ \tag{6}$$

$$\begin{aligned}\Pi_t\left(q_f,q_t\right)=&\,p_tE\min(q_t,D_t)+v_fq_f+w_fE\min\left(q_f,D_f\right)-g_tE(D_t-q_t)^+\\&-h_tE(q_t-D_t)^+-h_fE\left(q_f-D_f\right)^+-c_tq_t-c_fq_f\end{aligned} \tag{7}$$

Proposition 2. *When considering an option contract, there exists a situation where* $q_t^*=\underset{q_t\in S}{\arg}\max\Pi_t(q_t)$, *leading to the optimum empty container option ordering volume for the forwarder and the optimal empty container leasing volume for the carrier being* $\left(q_f^*,q_t^*\right)$.

Proof. By employing Leibniz's law, the derivatives of the profit function for the forwarder can be derived as follows:

$$\begin{aligned}\frac{\partial\Pi_f\left(q_f,q_t\right)}{\partial q_f}&=\left(p_f-w_f+g_f\right)\\&\cdot\left[\int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f}^{\infty}\int_0^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x\right]-v_f\end{aligned} \tag{8}$$

$$\begin{aligned}\frac{\partial^2\Pi_f\left(q_f,q_t\right)}{\partial q_f^2}&=-\left(p_f-w_f+g_f\right)\\&\cdot\left[\int_0^{q_t}f\left(q_f\right)g(y)\mathrm{d}y+\frac{1}{\lambda_t}\int_0^{q_f}f(x)g\left(-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t\right)\mathrm{d}x\right]\end{aligned} \tag{9}$$

Obviously, $\frac{\partial^2\Pi_f\left(q_f,q_t\right)}{\partial q_f^2}\leqslant 0$, indicating the existence of a value of q_f at which the profit function of forwarder Π_f reach its maximum value.

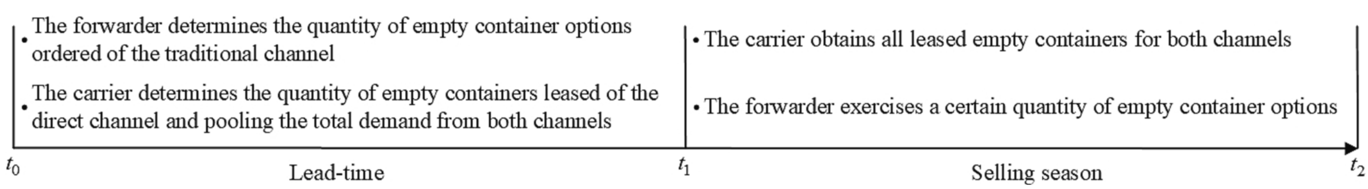

Fig. 4. The decision-making process of the forwarder and the carrier when introducing option contract.

Letting $\frac{\partial \Pi_f(q_f,q_t)}{\partial q_f}=0$ yields the optimal empty container option order quantity q_f^*. We employ $q_f^*=q_f(q_t)$ to denote the optimal response function of the forwarder. Consequently, the profit function of the carrier can be denoted as $\Pi_t\left(q_f,q_t\right)=\Pi_t\left(q_f(q_t),q_t\right)$ in this context. Taking the first-order derivative of $\Pi_t\left(q_f,q_t\right)$ with respect to q_t yields:

$$\frac{d\Pi_t\left(q_f,q_t\right)}{dq_t}=\frac{\partial \Pi_t\left(q_f,q_t\right)}{\partial q_f}\frac{dq_f}{dq_t}+\frac{\partial \Pi_t\left(q_f,q_t\right)}{\partial q_t} \tag{10}$$

From the equation above, it is evident that $\left.\frac{d\Pi_t\left(q_f,q_t\right)}{dq_t}\right|_{q_f=0}=p_t+g_t-c_t>0$ and $\left.\frac{d\Pi_t\left(q_f,q_t\right)}{dq_t}\right|_{q_f=\infty}=-h_t-c_t<0$. Furthermore, due to the continuity of the function $\frac{d\Pi_t\left(q_f,q_t\right)}{dq_t}$, it follows that at least one solution that results in $\frac{d\Pi_t\left(q_f,q_t\right)}{dq_t}=0$. Hence, there exists a value $q_t^*=\underset{q_t\in S}{\operatorname{argmax}}\Pi_t(q_t)$ such that both members possess an optimal set of empty container leasing quantities $\left(q_f^*,q_t^*\right)$. Here, S represents the set of q_t that satisfies the condition $\frac{d\Pi_t\left(q_f,q_t\right)}{dq_t}=0$. Q.E.D.

Corollary 1. For $\forall q_t$, it follows that $\frac{dq_f}{dq_t}<0$ and $\left|\frac{dq_f}{dq_t}\right|\leqslant\lambda_t$.

Corollary 1 demonstrates that in the event of an increase in the carrier's leasing quantity of empty containers within the direct channel, the forwarder will correspondingly reduce the optimal quantity of empty container options ordered in the traditional channel. Furthermore, the rate at which the option order quantity decreases is less than the rate at which customers transfer from the direct channel to the traditional channel, denoted as λ_t. Refer to Appendix A for proof.

Corollary 2. The forwarder's optimal number of empty container options ordered q_f^* is a monotonically decreasing function of its empty container option ordering price v_f and exercise price w_f and a monotonically increasing function of its unit canvassing price p_f.

Proof. For the forwarder the following equation always holds:

$$\frac{\partial \Pi_f}{\partial q_f^*}=\left(p_f-w_f+g_f\right)\cdot\left[\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f^*}^{\infty}\int_0^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x\right]-v_f=0 \tag{11}$$

Both sides of the above equation simultaneously solve for the derivatives of q_f^* with respect to the empty container option ordering price v_f, the empty container option exercise price w_f and the unit canvassing price p_f and we can then simplify the results to obtain:

$$\frac{\partial q_f^*}{\partial v_f}=-\frac{1}{\left(p_f-w_f+g_f\right)\left[\int_0^{q_t}f\left(q_f^*\right)g(y)\mathrm{d}y+\frac{1}{\lambda_t}\int_0^{q_f^*}f(x)g\left(-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t\right)\mathrm{d}x\right]}<0 \tag{12}$$

$$\frac{\partial q_f^*}{\partial w_f}=-\frac{\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f^*}^{\infty}\int_0^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x}{\left(p_f-w_f+g_f\right)\left[\int_0^{q_t}f\left(q_f^*\right)g(y)\mathrm{d}y+\frac{1}{\lambda_t}\int_0^{q_f^*}f(x)g\left(-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t\right)\mathrm{d}x\right]}<0 \tag{13}$$

$$\frac{\partial q_f^*}{\partial p_f}=\frac{\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f^*}^{\infty}\int_0^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x}{\left(p_f-w_f+g_f\right)\left[\int_0^{q_t}f\left(q_f^*\right)g(y)\mathrm{d}y+\frac{1}{\lambda_t}\int_0^{q_f^*}f(x)g\left(-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t\right)\mathrm{d}x\right]}>0 \tag{14}$$

Q.E.D.

Corollary 3. The optimal expected profit of the forwarder as a monotonically decreasing function of the empty container option ordering price v_f and exercise price w_f and as a monotonically increasing function of the unit canvassing price of the traditional channel p_f. The optimal expected profit of the carrier is simultaneously a monotonically increasing function of the empty container option ordering price, the empty container option execution price and the unit canvassing price of the direct channel.

Proof. The derivatives of the optimal expected profit for the forwarder with respect to the empty container option ordering price v_f, the empty container option exercise price w_f and the unit canvassing price of the traditional channel p_f can be obtained separately as:

$$\frac{\partial \Pi_f\left(q_f^*\right)}{\partial v_f}=-q_f^*<0 \tag{15}$$

$$\frac{\partial \Pi_f\left(q_f^*\right)}{\partial w_f}=-\int_0^{q_f^*}\int_0^{q_t}xf(x)g(y)\mathrm{d}y\mathrm{d}x-\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty}q_f^*f(x)g(y)\mathrm{d}y\mathrm{d}x-\int_{q_f^*}^{\infty}\int_0^{\infty}q_f^*f(x)g(y)\mathrm{d}y\mathrm{d}x-\int_0^{q_f^*}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}[x+\lambda_t(y-q_t)]f(x)g(y)\mathrm{d}y\mathrm{d}x<0 \tag{16}$$

$$\frac{\partial \Pi_f\left(q_f^*\right)}{\partial p_f}=\int_0^{q_f^*}\int_0^{q_t}xf(x)g(y)\mathrm{d}y\mathrm{d}x+\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty}q_f^*f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f^*}^{\infty}\int_0^{\infty}q_f^*f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_0^{q_f^*}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}[x+\lambda_t(y-q_t)]f(x)g(y)\mathrm{d}y\mathrm{d}x>0 \tag{17}$$

The proof of the carrier resembles that of the forwarder and will not be repeated here. Q.E.D.

Corollary 4. Option contract can make risks shared between the carrier and the forwarder, effectively enhancing the revenue of the forwarder.

Proof. Combining equations *(1) and (6)*, we can get the profit difference of the forwarder before and after the use of option contract is $\Delta\Pi_f^{oc}=-w_fE\min\left(q_f,D_f\right)-v_fq_f+wq_f+h_fE\left(q_f-D_f\right)^+$. Combined with $v_f+w_f<w$, it is evident that $v_fq_f+w_fE\min\left(q_f,D_f\right)\leqslant\left(v_f+w_f\right)q_f<wq_f$. Thus, it follows that $\Delta\Pi_f^{oc}>0$, indicating that the option contract can effectively enhance the profitability of the forwarder. Q.E.D.

5. Centralised model

To provide a benchmark for the coordination mechanism of the system, this section discusses the model for empty container and empty container option decisions in a DCCTSC with consideration of option contract under a centralised mode. Due to the fact that both members only consider their own marginal benefits in decision-making, double marginalisation and inter-channel conflict issues become prominent. As a result, under the decentralised mode, the overall revenue of the container transportation service chain cannot achieve a systematically optimal level. The centralised mode can also be called the noncompetitive mode, which means that the carrier and the forwarder are combined into one decision-maker with the right to make decisions on the ordering of empty container options in the traditional channel and the leasing of empty containers in the direct channel. Its objective is to maximise the desired profitability of the overall system of the DCCTST, thus achieving the optimal level of the system. In a centralised mode of introducing an option contract, the income of the whole system is mainly derived from the actual selling price of empty containers in both channels. The costs mainly include the empty container rental cost of two channels, the cost of shortage of containers when demand exceeds supply and the storage fee of empty containers when supply exceeds demand. The total system profit of the DCCTSC can therefore be articulated as:

$$\begin{aligned}\Pi_c\left(q_f,q_t\right) &= p_f E\min\left(q_f,D_f\right) - g_f E\left(D_f-q_f\right)^+ + p_t E\min\left(q_t,D_t\right)\\ &-g_t E\left(D_t-q_t\right)^+ - h_f E\left(q_f-D_f\right)^+ - h_t E\left(q_t-D_t\right)^+ - c_f q_f - c_t q_t\end{aligned} \tag{18}$$

Proposition 3. *The total profit of the centralised DCCTSC is a continuously differentiable joint concave function with respect to q_f and q_t. There exists a sole $\left(q_f^{c*},q_t^{c*}\right)$ that maximises the total profit of the DCCTSC under centralised decision-making.*

Proof. Calculating the first-order derivatives with respect to q_f and q_t separately for the total profit function $\Pi_c\left(q_f,q_t\right)$ yields the following:

$$\begin{aligned}\frac{\partial\Pi_c\left(q_f,q_t\right)}{\partial q_f} &= \left(p_f+g_f\right)\left[\int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right]\\ &-\lambda_f p_t\int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x + \lambda_f g_t\left[\int_{q_f+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x\right.\\ &\left.+\int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t}^{q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{\infty}\int_{q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right]\\ &-h_f\int_0^{q_f}\int_0^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x - \lambda_f h_t\int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x - c_f\end{aligned} \tag{19}$$

$$\begin{aligned}\frac{\partial\Pi_c\left(q_f,q_t\right)}{\partial q_t} &= \left(p_t+g_t\right)\left[\int_0^{\infty}\int_{q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t}^{q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x\right.\\ &\left.+\int_{q_f+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x\right] - \lambda_t\left(p_f+h_f\right)\int_0^{q_f}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x\\ &+\lambda_t g_f\left[\int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{\infty}\int_{q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right]\\ &-h_t\left[\int_0^{q_f}\int_0^{q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x\right]\end{aligned} \tag{20}$$

Next, the calculation shows that $\frac{\partial^2\Pi_c\left(q_f,q_t\right)}{\partial q_f^2}<0$, $\frac{\partial^2\Pi_c\left(q_f,q_t\right)}{\partial q_t^2}<0$ and $\frac{\partial^2\Pi_c\left(q_f,q_t\right)}{\partial q_f^2}\cdot\frac{\partial^2\Pi_c\left(q_f,q_t\right)}{\partial q_t^2}-\frac{\partial^2\Pi_c\left(q_f,q_t\right)}{\partial q_f\partial q_t}\cdot\frac{\partial^2\Pi_c\left(q_f,q_t\right)}{\partial q_t\partial q_f}>0$. Hence, there exists a set of values $\left(q_f^{c*},q_t^{c*}\right)$ comprising the optimal number of empty container options ordered by the forwarder and the optimal number of empty containers leased by the carrier, such that the profit of the dual-channel system in the noncompetitive mode is maximised. This set of optimal empty container decisions $\left(q_f^{c*},q_t^{c*}\right)$ satisfies $\frac{\partial\Pi_c\left(q_f,q_t\right)}{\partial q_f}=0$ and $\frac{\partial\Pi_c\left(q_f,q_t\right)}{\partial q_t}=0$. Proof see Appendix. Q.E.D.

Corollary 5. In the centralised mode, the equation $\Pi_c\left(q_f^{c*},q_t^{c*}\right)\geqslant\Pi_f\left(q_f^{*},q_t^{*}\right)+\Pi_t\left(q_f^{*},q_t^{*}\right)$ consistently holds true.

Proof. $\left(q_f^{c*},q_t^{c*}\right)$ is a set of critical decision values for both the forwarder and the carrier that maximise the total profit of the dual-channel system and substituting them into Eq. *(18)* readily yields $\Pi_f\left(q_f^{c*},q_t^{c*}\right)+\Pi_t\left(q_f^{c*},q_t^{c*}\right)=\Pi_c\left(q_f^{c*},q_t^{c*}\right)$. Furthermore, from Proposition 3, we can deduce that Π_c is a strictly concave function, and $\left(q_f^{*},q_t^{*}\right)$ forms a distinct set of values from $\left(q_f^{c*},q_t^{c*}\right)$. Consequently, the equation $\Pi_c\left(q_f^{c*},q_t^{c*}\right)\geqslant\Pi_f\left(q_f^{*},q_t^{*}\right)+\Pi_t\left(q_f^{*},q_t^{*}\right)$ must hold. Q.E.D.

The conditions for $\Pi_f\left(q_f^{c*},q_t^{c*}\right)+\Pi_t\left(q_f^{c*},q_t^{c*}\right)=\Pi_c\left(q_f^{c*},q_t^{c*}\right)$ to hold are $q_f^{*}=q_f^{c*}$ and $q_t^{*}=q_t^{c*}$. The implication of this expression is that when the forwarder and the carrier are in a competitive state, if the quantity of leased empty containers by the carrier and the quantity of options ordered by the forwarder cannot simultaneously reach the level observed in the centralised mode, the combined profits of both parties will invariably be lower than the total system profit achieved under the centralised mode. To explain further, if both entities can collaboratively decide to align the quantity of containers leased by the carrier and the quantity of option contracts procured by the freight forwarder with the optimal inventory levels observed in the centralised mode, then their combined profits can ascend to the maximum profit level witnessed under the centralised mode.

6. The coordination model with improved revenue-sharing contract

In the above chapter, we applied the option contract to the DCCTSC, where the carrier assumes part of the risk posed by uncertain demand for the forwarder. To compensate for the losses suffered by the carrier in taking the risk, we introduce an improved revenue-sharing contract, whereby both the forwarder and the carrier share a part of their proceeds, making the design of the contract for the whole system more rational and effective. The combination of the improved revenue-sharing contract and the option contract compensates for the shortcomings of a separate option contract. The option contract can increase the profitability of the entire dual-channel transportation system and the improved revenue-sharing contract can overcome the double marginal effect.

We define the improved revenue-sharing option contract as $\{v_f,w_f,\varphi_1,\varphi_2\}$. Among them, φ_1 denotes the traditional channel sharing factor, i.e., the proportion of traditional channel revenue that the forwarder can obtain at the conclusion of the selling season. Similarly, φ_2 signifies the direct channel sharing factor, i.e., the proportion of direct channel revenue that the carrier can obtain at the conclusion of the selling season. The execution mechanism of this contract can be summarised as follows. Firstly, the forwarder and the carrier make decisions on the optimal option ordering quantity q_f^{c*} and the optimal empty container leasing quantity q_t^{c*}, respectively, based on Equations (19) and (20); Secondly, substituting the values of q_f^{c*} and q_t^{c*} into Equations (24) and (25) to obtain expressions that relate to the four contract parameters. Subsequently, both members engage in negotiation to determine the values of φ_1 and w_f, ensuring a fair allocation of profits. Third, the forwarder orders empty container options q_f from the carrier in advance of the selling season at a price of v_f, and the carrier leases empty containers from the container leasing company at a volume of q_f+q_t. Fourth, the forwarder exercises the required empty container options at the beginning of the selling season at the price of w_f based on actual market demand. Finally, at the end of the selling season, the carrier shares $1-\varphi_2$ of the revenue from the direct channel with the forwarder, and the forwarder shares $1-\varphi_1$ of the revenue from the traditional channel with the carrier. In this case, the expected profit of the forwarder is:

$$\Pi_f^s(q_f,q_t) = \varphi_1 p_f E\min(q_f,D_f) + (1-\varphi_2)p_t E\min(q_t,D_t) - v_f q_f - w_f E\min(q_f,D_f) - g_f E(D_f-q_f)^+ \tag{21}$$

It is clear from the calculation that when $\varphi_1 p_f - w_f + g_f - (1-\varphi_2)\lambda_f p_t > 0$, there is also $\frac{\partial^2\Pi_f^s(q_f,q_t)}{\partial q_f^2} \leqslant 0$. At this juncture, there exists an optimal number of empty container options ordered, denoted as q_f^{s*}, which maximises the profit for the forwarder. This particular q_f^{s*} should satisfy the condition where $\frac{\partial\Pi_f^s(q_f,q_t)}{\partial q_f}=0$. We define $q_f^{s*}=q_f(q_t)$ as the optimal response function for the forwarder, so that the profit for the carrier under the improved revenue-sharing option contract coordination is $\Pi_t^s(q_f,q_t) = \Pi_t^s(q_f(q_t),q_t)$, which can be expressed in the following form:

$$\Pi_t^s(q_f(q_t),q_t) = \varphi_2 p_t E\min(q_t,D_t) + (1-\varphi_1)p_f E\min(q_f(q_t),D_f) + v_f q_f(q_t) + w_f E\min(q_f(q_t),D_f) - g_t E(D_t-q_t)^+ - h_t E(q_t-D_t)^+ - h_f E(q_f(q_t)-D_f)^+ - c_t q_t - c_f q_f(q_t) \tag{22}$$

In accordance with the principle of the master–slave strategy, the optimum volume of empty containers leased q_t^{s*} by the carrier satisfies the following equation:

$$\frac{d\Pi_t^s(q_f,q_t)}{dq_t} = \frac{\partial\Pi_t^s(q_f,q_t)}{\partial q_f}\frac{dq_f}{dq_t} + \frac{\partial\Pi_t^s(q_f,q_t)}{\partial q_t} = 0 \tag{23}$$

We let $q_f^{s*}=q_f^{c*}$ and $q_t^{s*}=q_t^{c*}$, which implies that after contractual coordination, the optimum number of empty containers rented by the carrier and the optimum number of empty container options ordered by the forwarder both attain the values under the centralised mode. Since the optimal level needs to meet the first-order conditions $\frac{\partial\Pi_c(q_f,q_t)}{\partial q_f}=0$ and $\frac{\partial\Pi_f^s(q_f,q_t)}{\partial q_f}=0$, and since $\frac{\partial\Pi_c(q_f,q_t)}{\partial q_f} = \frac{\partial\Pi_f^s(q_f,q_t)}{\partial q_f} + \frac{\partial\Pi_t^s(q_f,q_t)}{\partial q_f}$, we have $\frac{\partial\Pi_t^s(q_f,q_t)}{\partial q_f}=0$, which further leads to $\frac{d\Pi_t^s(q_f,q_t)}{dq_t} = \frac{\partial\Pi_t^s(q_f,q_t)}{\partial q_t}$. Thus, under the revenue-sharing option contract, the combination of the optimum number of empty containers option ordered by the forwarder and the optimum volume of empty containers leased by the carrier (q_f^{s*},q_t^{s*}) meets $\frac{\partial\Pi_f^s(q_f,q_t)}{\partial q_f}=0$ and $\frac{\partial\Pi_t^s(q_f,q_t)}{\partial q_t}=0$.

Proposition 4. *In the context of the improved revenue-sharing option contract, if the DCCTSC wants to attain coordination, it is imperative that the contract parameters $\{v_f,w_f,\varphi_1,\varphi_2\}$ need to satisfy the subsequent formulas:*

$$v_f = \frac{[p_t+g_t-c_t-(p_t+g_t+h_t)\beta_1]\cdot\left[(\varphi_1 p_f - w_f + g_f)\beta_2 - g_f\int_{q_t}^{\infty}g(y)dy\right]}{(1-\beta_1)\left[(p_f+g_f+h_f)\beta_2 - g_f\int_{q_t}^{\infty}g(y)dy\right]} \cdot\frac{\alpha_2[p_f+g_f-c_f-(p_f+g_f+h_f)\alpha_1]}{(p_t+g_t+h_t)\alpha_2 - g_t\int_{q_f}^{\infty}f(x)dx} - (\varphi_1 p_f - w_f + g_f)\cdot(1-\alpha_1) \tag{24}$$

$$\varphi_2 = \frac{[p_t+g_t-c_t-(p_t+g_t+h_t)\beta_1]\cdot[(1-\varphi_1)p_f+w_f+h_f]\beta_2 + \left[(p_f+g_f+h_f)\beta_2 - g_f\int_{q_t}^{\infty}g(y)dy\right]\cdot[(g_t+h_t)\beta_1 - g_t + c_t]}{p_t(1-\beta_1)\left[(p_f+g_f+h_f)\beta_2 - g_f\int_{q_t}^{\infty}g(y)dy\right]} \tag{25}$$

Proof. For the purpose of deeper analysis, this paper assumes:

$$\alpha_1 = \int_0^{q_f}\int_0^{q_t} f(x)g(y)dydx + \int_0^{q_f}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t} f(x)g(y)dydx$$

$$\alpha_2 = \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t} f(x)g(y)dydx$$

$$\beta_1 = \int_0^{q_f}\int_0^{q_t} f(x)g(y)dydx + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t} f(x)g(y)dydx$$

$$\beta_2 = \int_0^{q_f}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t} f(x)g(y)dydx$$

Thus, Equations (19) and (20), as well as $\frac{\partial\Pi_f^s(q_f,q_t)}{\partial q_f}$ and $\frac{\partial\Pi_t^s(q_f,q_t)}{\partial q_t}$ can be expressed as:

$$\frac{\partial\Pi_c(q_f,q_t)}{\partial q_f} = p_f+g_f-c_f+\lambda_f g_t\int_{q_f}^{\infty}f(x)dx - (p_f+g_f+h_f)\alpha_1 - \lambda_f(p_t+g_t+h_t)\alpha_2 \tag{26}$$

$$\frac{\partial\Pi_c(q_f,q_t)}{\partial q_t} = p_t+g_t-c_t+\lambda_t g_f\int_{q_t}^{\infty}g(y)dy - (p_t+g_t+h_t)\beta_1 - \lambda_t(p_f+g_f+h_f)\beta_2 \tag{27}$$

$$\frac{\partial\Pi_f^s(q_f,q_t)}{\partial q_f} = (\varphi_1 p_f - w_f + g_f)\cdot(1-\alpha_1) - \lambda_f(1-\varphi_2)p_t\alpha_2 - v_f \tag{28}$$

$$\frac{\partial\Pi_t^s(q_f,q_t)}{\partial q_t} = (\varphi_2 p_t + g_t)\cdot(1-\beta_1) - \lambda_t[(1-\varphi_1)p_f + w_f + h_f]\beta_2 - h_t\beta_1 - c_t \tag{29}$$

Letting Equations (26) ~ (29) equal to 0 yields the following equation:

$$\lambda_f = \frac{p_f+g_f-c_f-(p_f+g_f+h_f)\alpha_1}{(p_t+g_t+h_t)\alpha_2 - g_t\int_{q_f}^{\infty}f(x)dx} \tag{30}$$

$$\lambda_t = \frac{p_t+g_t-c_t-(p_t+g_t+h_t)\beta_1}{(p_f+g_f+h_f)\beta_2 - g_f\int_{q_t}^{\infty}g(y)dy} \tag{31}$$

$$v_f = \lambda_f(1-\varphi_2)p_t\alpha_2 - (\varphi_1 p_f - w_f + g_f)\cdot(1-\alpha_1) \tag{32}$$

$$\varphi_2 = \frac{\lambda_t[(1-\varphi_1)p_f + w_f + h_f]\beta_2 + (g_t+h_t)\beta_1 - g_t + c_t}{p_t(1-\beta_1)} \tag{33}$$

Proposition 4 can be obtained by combining Equations (30) ~ (33). The proof process is detailed in the Appendix. Q.E.D.

Proposition 4 illustrates that under the improved revenue-sharing option contract, if four contract parameters comply with the corresponding relationships in Equations (24) and (25), then the optimal empty container option ordering volume by the forwarder and the optimum volume of empty containers leased by the carrier can both attain their respective optimal values under centralised decision-making and realise system coordination. Additionally, due to the one-to-one linear relationships among the four parameters, where each parameter can be expressed mathematically in terms of the other two, fine-tuning individual parameters allows us to achieve an equitable distribution of profits within this dual-channel system. For instance, taking $\{w_f,\varphi_1\}$ and $\{v_f,\varphi_2\}$ as two corresponding sets of parameters, the carrier only needs to adjust the option exercise price v_f and the traditional channel sharing factor φ_2. This adjustment allows for the determination of two key parameters: the option order price w_f and the direct channel sharing factor φ_1, thus effortlessly partitioning the profits between the two members.

7. Pareto improvement analysis

While Proposition 4 ensures that the forwarder and the carrier reach a coordinated state, achieving the overall profit equal to the optimal value under centralised decision-making, it does not guarantee that both members will experience improved profits compared to decentralised decisions. Therefore, to render the proposed contract executable, Proposition 5 highlights the Pareto-improvement interval, ensuring that within this interval, both members can attain a mutually beneficial outcome.

Proposition 5. *When coordinating the DCCTSC system using the improved revenue-sharing option contract, given the option execution price w_f, the profits of both members in the dual-channel system can achieve Pareto improvement in the following two cases when $\varphi_1 \in [\underline{\varphi_1}, \overline{\varphi_1}]$: (1) $\frac{\partial \Delta \Pi_f}{\partial \varphi_1} > 0$, $\underline{\varphi_1} = \max\left(0, \varphi_1^f\right)$ and $\overline{\varphi_1} = \min(\varphi_1^t, 1)$ ($\underline{\varphi_1} < \overline{\varphi_1}$); (2) $\frac{\partial \Delta \Pi_f}{\partial \varphi_1} < 0$, $\underline{\varphi_1} = \max(0, \varphi_1^t)$ and $\overline{\varphi_1} = \min\left(\varphi_1^f, 1\right)$ ($\underline{\varphi_1} < \overline{\varphi_1}$). Wherein, φ_1^f and φ_1^t are the values of φ_1 corresponding to $\Delta\Pi_f = 0$ and $\Delta\Pi_t = 0$, respectively.*

Proof. Substituting the optimum option ordering quantity for the forwarder $q_f^{c^*}$ and the optimum empty container leasing quantity for the carrier $q_t^{c^*}$ under centralised decision-making into Equations (21) and (22) yields the coordinated revenues, $\Pi_f^s\left(q_f^{c^*}, q_t^{c^*}\right)$ and $\Pi_t^s\left(q_f^{c^*}, q_t^{c^*}\right)$, for both parties. Furthermore, their revenues under decentralised decision-making are recorded as Π_f^* and Π_t^*, respectively. Therefore, the difference in profits for these two members before and after coordination can be represented as $\Delta\Pi_f = \Pi_f^s\left(q_f^{c^*}, q_t^{c^*}\right) - \Pi_f^*$ and $\Delta\Pi_t = \Pi_t^s\left(q_f^{c^*}, q_t^{c^*}\right) - \Pi_t^*$, respectively, as shown in the following equations. The mathematical representation of both members achieving a Pareto improvement in profits is expressed as $\Delta\Pi_f \geqslant 0$ and $\Delta\Pi_t \geqslant 0$.

$$\Delta\Pi_f = (\varphi_1 - 1)p_f E\min\left(q_f, D_f\right) + (1 - \varphi_2)p_t E\min(q_t, D_t) \tag{34}$$

$$\Delta\Pi_t = (\varphi_2 - 1)p_t E\min(q_t, D_t) + (1 - \varphi_1)p_f E\min\left(q_f, D_f\right) \tag{35}$$

Calculating the first-order derivative of these two profit difference functions with respect to φ_1 yields the following:

$$\frac{\partial \Delta\Pi_f}{\partial \varphi_1} = p_f E\min\left(q_f, D_f\right) + \frac{[p_t + g_t - c_t - (p_t + g_t + h_t)\beta_1] \cdot p_f \beta_2 \cdot E\min(q_t, D_t)}{(1 - \beta_1)\left[\left(p_f + g_f + h_f\right)\beta_2 - g_f \int_{q_t}^{\infty} g(y)\mathrm{d}y\right]} \tag{36}$$

$$\frac{\partial \Delta\Pi_t}{\partial \varphi_1} = \frac{-[p_t + g_t - c_t - (p_t + g_t + h_t)\beta_1] \cdot p_f \beta_2 \cdot E\min(q_t, D_t)}{(1 - \beta_1)\left[\left(p_f + g_f + h_f\right)\beta_2 - g_f \int_{q_t}^{\infty} g(y)\mathrm{d}y\right]} - p_f E\min\left(q_f, D_f\right) \tag{37}$$

Evidently, $\frac{\partial \Delta\Pi_f}{\partial \varphi_1} = -\frac{\partial \Delta\Pi_t}{\partial \varphi_1}$. Thus, when $\frac{\partial \Delta\Pi_f}{\partial \varphi_1} > 0$, it necessarily implies that $\frac{\partial \Delta\Pi_t}{\partial \varphi_1} < 0$, meaning that $\Delta\Pi_f$ increases with the increase of φ_1 and $\Delta\Pi_t$ decreases with the increase of φ_1. Additionally, since φ_1^f and φ_1^t must both satisfy the conditions of being greater than or equal to 0 and less than or equal to 1, taking $\underline{\varphi_1} = \max\left(0, \varphi_1^f\right)$, $\overline{\varphi_1} = \min(\varphi_1^t, 1)$ and $\underline{\varphi_1} < \overline{\varphi_1}$ results in the four scenarios shown in Fig. 5. At this juncture, $0 \leqslant \varphi_1^f < \varphi_1^t \leqslant 1$. When $\varphi_1 \in [\underline{\varphi_1}, \overline{\varphi_1}]$, the profit differences between the post-coordination and decentralised decision-making is greater than 0 for both the carrier and the forwarder, signifying that the profits of both members have undergone Pareto improvement. Likewise, when $\Delta\Pi_f$ decreases with the increase in φ_1, $\Delta\Pi_t$ will inevitably increase with the increase in φ_1. In this case, taking $\underline{\varphi_1} = \max(0, \varphi_1^t)$, $\overline{\varphi_1} = \min\left(\varphi_1^f, 1\right)$ and $\underline{\varphi_1} < \overline{\varphi_1}$ results in the four scenarios shown in Fig. 6. Accordingly, when $\varphi_1 \in [\underline{\varphi_1}, \overline{\varphi_1}]$, $\Delta\Pi_f > 0$ and $\Delta\Pi_t > 0$ hold, and both members

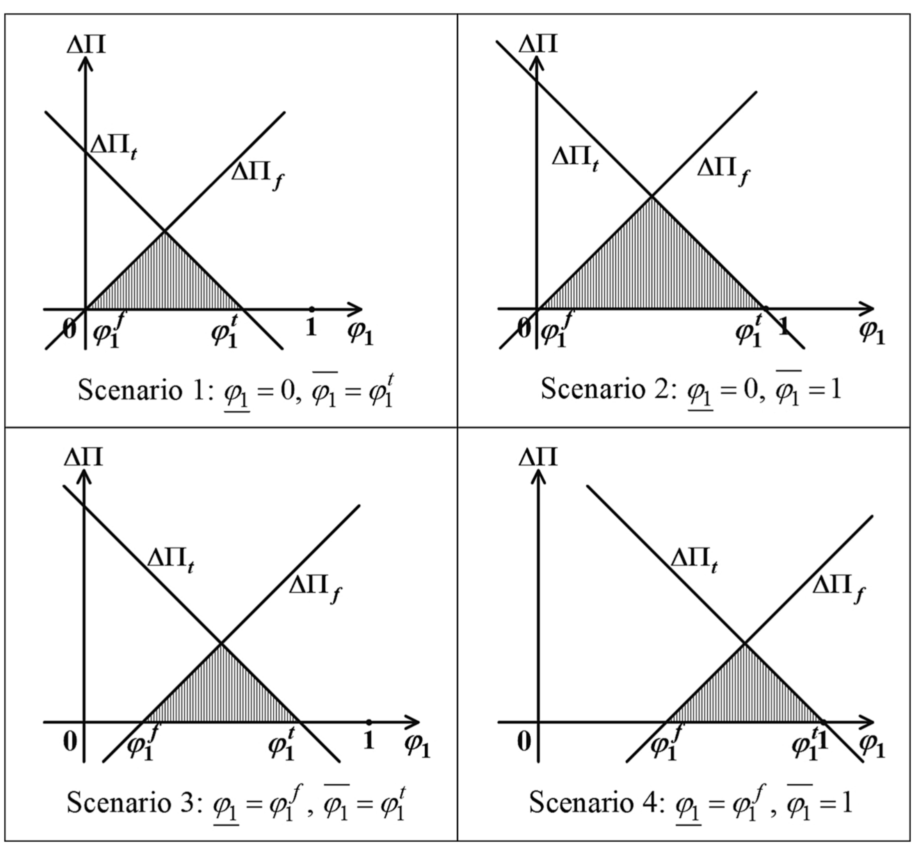

Fig. 5. The four Pareto improvement scenarios for $\frac{\partial \Delta\Pi_f}{\partial \varphi_1} > 0$.

achieve a Pareto improvement. Q.E.D.

Proposition 5 demonstrates that under the revenue-sharing option contract, when four parameters meet certain conditions, the profits of each member of the container transportation service chain are improved compared to the decentralised mode. This indicates that the contract can achieve conditional Pareto improvement, which facilitates the acceptance of the contract by the two members and its implementation. When the option exercise price is given, the forwarder can adjust the benefit distribution between the members of the DCCTSC by changing the proportion of the traditional channel revenue-sharing to create a win–win cooperation situation between two members.

8. Numerical examples

To interpret the findings of the previous sections more intuitively and to verify the effectiveness of the revenue-sharing option contract for the coordination of DCCTSC, this section will analyse them through specific arithmetic examples. The premise for this case study is that the demand in the traditional and direct channels follows a normal distribution, where $x \sim N(600, 400)$ and $y \sim N(500, 400)$. The primary parameters are configured as: $p_f = 1000$, $h_f = 100$, $c_f = 200$, $g_f = 100$, $\lambda_f = 0.6$, $p_t = 1000$, $h_t = 100$, $c_t = 200$, $g_t = 100$, and $\lambda_t = 0.3$. The values of these parameters are taken with reference to Liu et al. [12] and Luo et al. [3], with appropriate modifications to match the specifics of our study.

First, we verify the impact of the option contract on each member of the DCCTSC; that is, we investigate what happens to the forwarder's option ordering decisions, the carrier's empty container leasing decisions, and the profits of the forwarder and carrier when the option ordering price and executive price in the traditional channel are constantly changing. The results are shown in Fig. 7 and Fig. 8. As seen in Figs. 7 and 8, the trends in empty container decisions and profits for the forwarder are similar, and the carrier is in line with this law. When the unit option exercise price remains constant, as the unit option ordering price continues to rise, both the profit of the forwarder and the volume of their empty container options ordered continue to fall. Conversely, the profit of the carrier and the volume of their empty containers leased continue to rise. Similarly, when the unit option ordering price remains unchanged, as the option exercise price rises, the forwarder's profit and the number of empty container options ordered decrease. In contrast, the carrier's profit and the volume of empty containers leased increase. This is because when the order price or the exercise price of the option increases, the forwarder will order fewer empty container options or execute fewer empty container options due to the high cost of stocking them. They will therefore be able to meet less market demand, resulting in lower profits. When the market demand met by the forwarders decreases, some of the unmet customers of the traditional channel may move to the direct channel. The market demand that the carrier needs to meet increases accordingly, and the carrier will reserve more empty containers to cope with the expanding market; thus, the profits of the carrier will increase.

We then validated the effectiveness of the revenue-sharing option contract for the coordination of the DCCTSC. Given the mutual constraints and clear correspondences among the contract parameters v_f, w_f, φ_1 and φ_2, i.e., any one of them can be represented by the other two, it is possible to partition these four parameters into two groups, where one group of parameters is represented in terms of the other group. This facilitates an analysis of the relationships between the contract parameters. For instance, by referencing Equations (25) and (26), we express v_f and φ_2 in terms of w_f and φ_1. Subsequently, we analyse the relationships between w_f and φ_1 with respect to both v_f and φ_2, with the results depicted in Figs. 9 and 10.

Figs. 9 and 10 conduct sensitivity analysis regarding the option exercise price w_f and the sharing coefficient φ_1, simultaneously validating

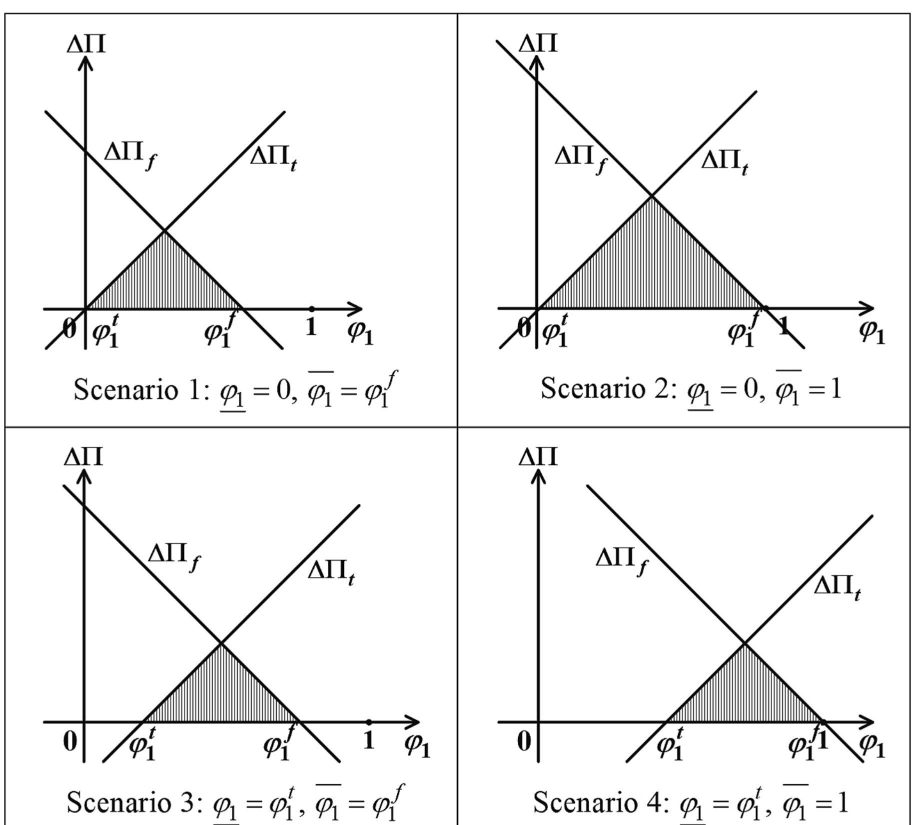

Fig. 6. The four Pareto improvement scenarios for $\frac{\partial \Delta\Pi_f}{\partial \varphi_1} < 0$.

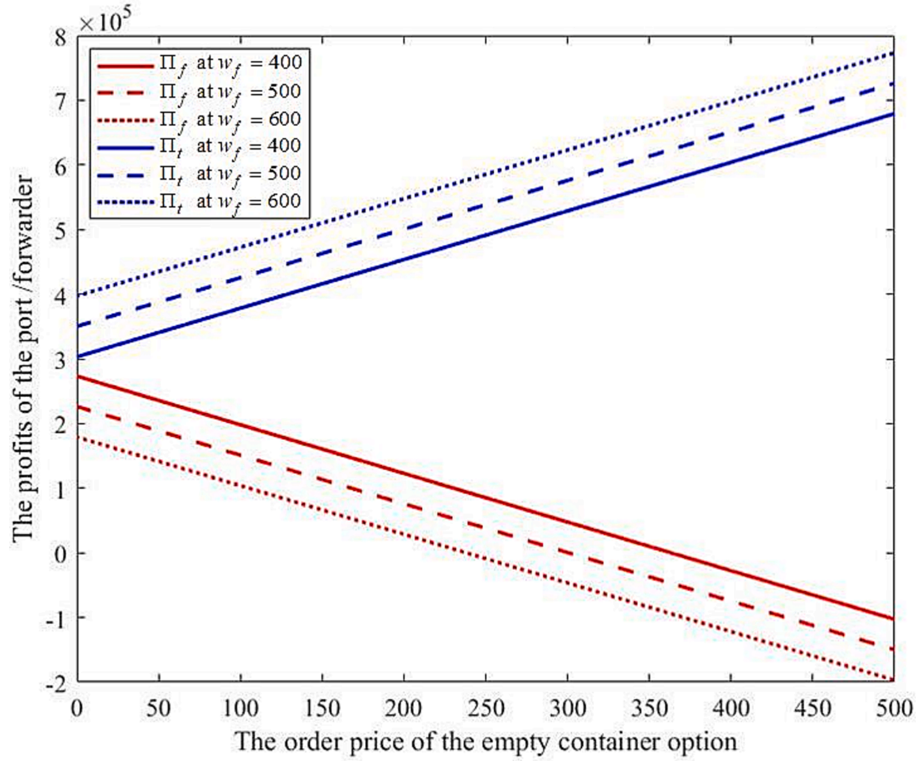

Fig. 7. The relationship between the profits of the two members and v_f/w_f.

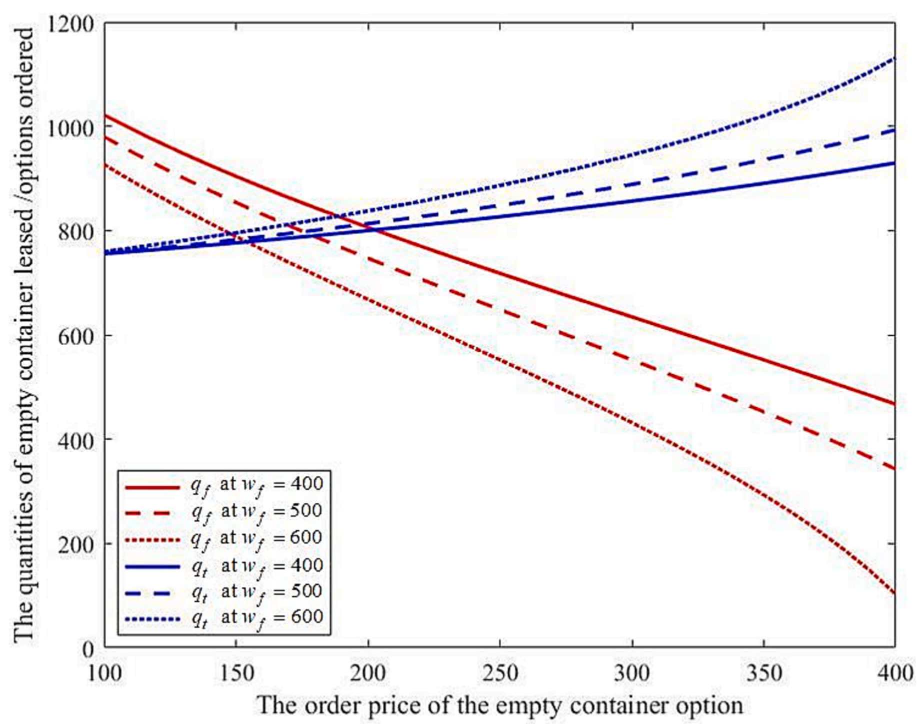

Fig. 8. The relationship between the empty container decisions of the two members and v_f/w_f.

Proposition 4. As shown in Figs. 9 and 10, when fixing the empty container option exercise price w_f, as φ_1 increases, the empty container option order price v_f tends to decrease, and the sharing coefficient φ_2 also tends to decrease. The underlying reason for this phenomenon lies in the increase in the sharing coefficient φ_1 amplifies the marginal profit of the forwarder while diminishing that of the carrier. This, in turn, results in an escalation in the procurement of options by the forwarder and a decreased leasing of empty containers by the carrier. For the purpose of sustaining the dual-channel system in a harmonised state, the carrier typically realigns the empty container decisions between the two channels by actively lowering option purchase prices and diminishing their share of profit from the direct sales channel. This realignment

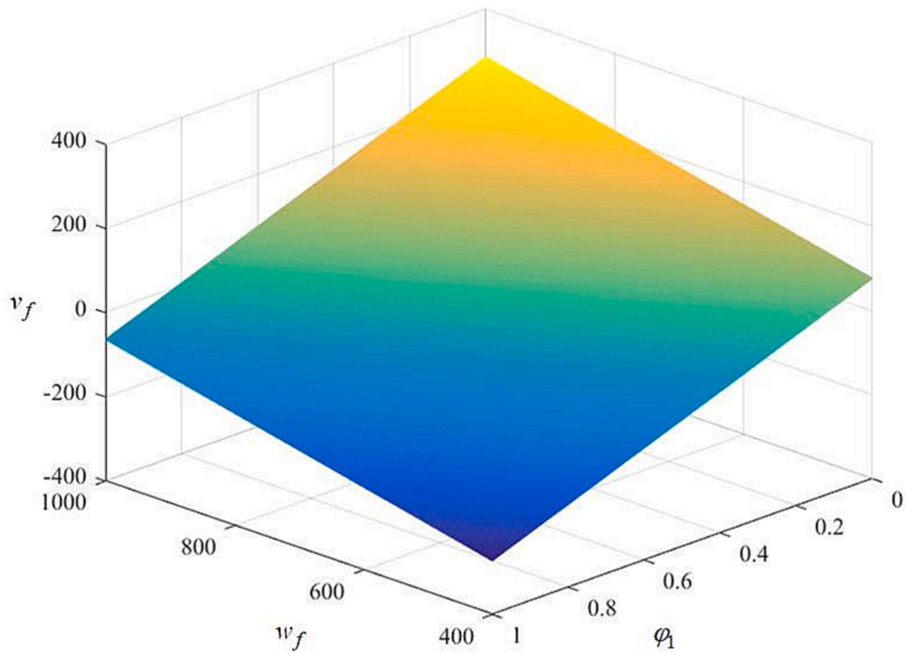

Fig. 9. The relationship between contract parameters w_f, φ_1 and v_f.

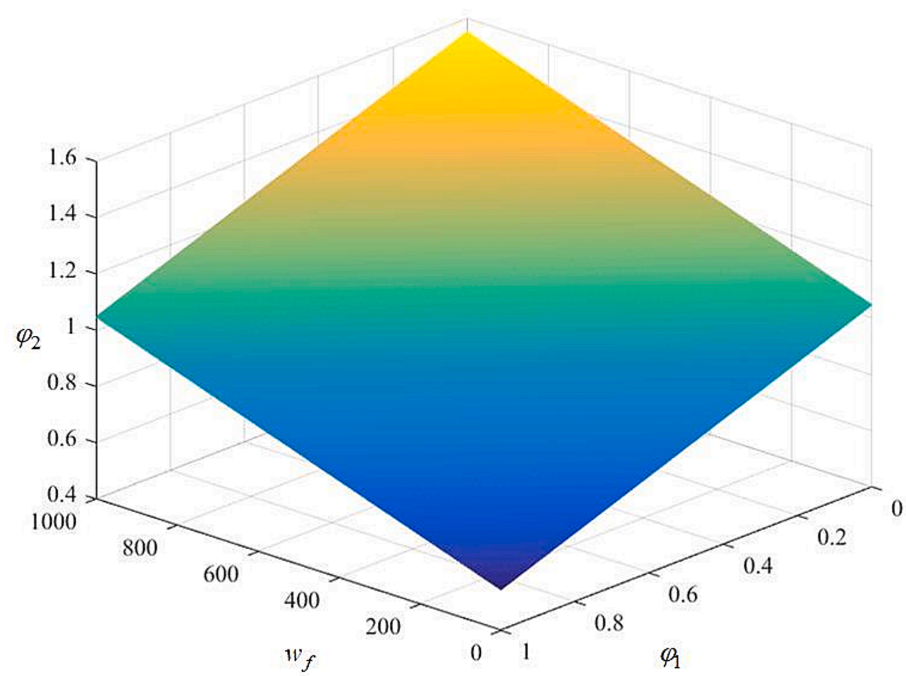

Fig. 10. The relationship between contract parameters w_f, φ_1 and φ_2.

continues until both channels' empty container decisions return to their ideal levels. Analogously, when φ_1 is held constant, both the empty container option order price v_f and the sharing coefficient φ_2 show an increasing trend as w_f increases. This is also attributed to the increase in the option exercise price w_f, which reduces the marginal profit for the forwarder while enhancing it for the carrier. Consequently, this decrease in the forwarder's option orders and the increase in the carrier's empty container leasing are observed. Likewise, to keep coordination in the dual-channel system, the carrier, on one hand, elevates the option purchase prices and, on the other hand, augments their profit-sharing ratio from the direct sales channel, thus readjusting the decisions in both channels to the ideal levels.

Subsequently, we proceed to analyse the influence of w_f and φ_1 on the profitability of the forwarder, the carrier and the whole system. The results are illustrated in Fig. 11.

As depicted in the figure above, even though the respective profits of both the forwarder and the carrier vary with changes in the empty container option exercise price w_f and the sharing factor φ_1, the overall profit of this dual-channel system consistently remains at its optimum value under coordination, reaching the peak performance of profit achieved in the centralised mode, which is 576,380. Put differently, when coordinating the system using the improved revenue-sharing option contract, the regulation of w_f and φ_1 serves solely as a "profit

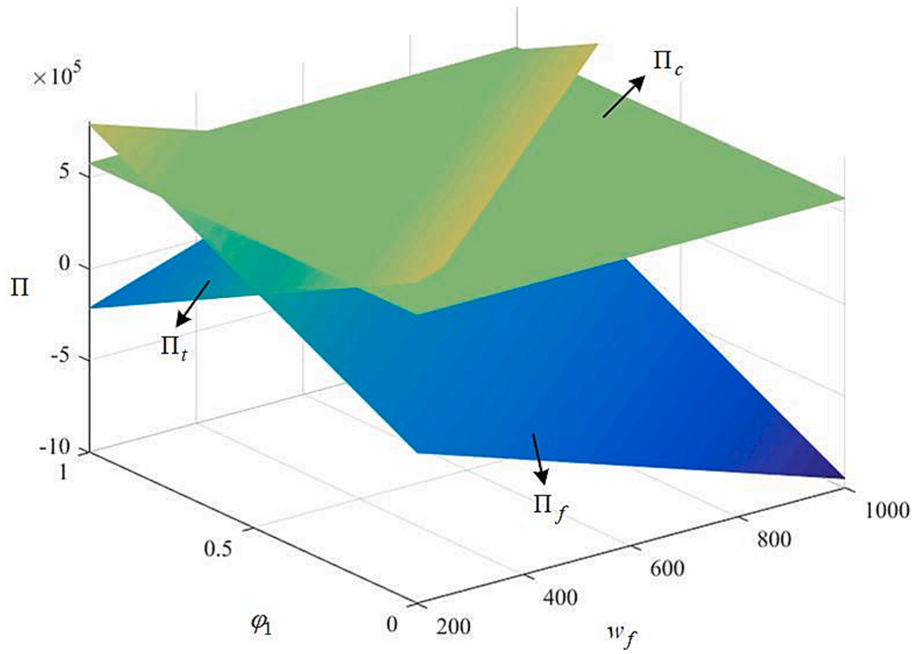

Fig. 11. The influence of changes in contractual parameters on the profits of the forwarder, the carrier, and the whole system.

redistribution" mechanism between the forwarder and the carrier, with no bearing on the coordinated state of the dual-channel system. In real-world markets, the values of parameters are typically closely tied to the bargaining power of the two members.

Taking the option contract parameters $v_f = 100$, $w_f = 500$, the wholesale price of empty containers for the forwarder when the option contract is not considered $w = 650$, while keeping other parameters constant, we can derive the ideal empty container decisions for both the forwarder and the carrier under different environments, as well as the corresponding peak profits, as presented in Table 6. Compared to the baseline model without an option contract, after introducing the option contract, the carrier's profit level declines by approximately 25.10 % due to sharing some of the risks on behalf of the forwarder. However, the forwarder's profit rises by approximately 16 times. The magnitude of the increase in the forwarder's profit far surpasses the decrease in the carrier's profit, which results in an overall profit improvement of approximately 3.49 % for the entire dual-channel system compared to the previous system.

Eventually, we investigate the parameter intervals associated with both participating members achieving Pareto improvements in profit within the dual-channel system, as depicted in Fig. 12.

From Fig. 12, it can be observed that when the combination coordinates $\{w_f, \varphi_1\}$ of option exercise prices and traditional channel profit-sharing coefficients fall within the shaded region, the dual-channel system attains coordination, and both the forwarder and the carrier can attain Pareto improvements. The parameter combinations that satisfy the two conditions exhibit flexibility, implying that each coordinate within the shaded region corresponds to a set of coordination parameters that can result in varying degrees of profit improvement for the participating members compared to their profits before coordination. For example, when $w_f = 500$ and $\varphi_1 = 0.66$, the profit of the forwarder is 171,240, the profit of the carrier is 405,140, and the aggregate profit of the dual-channel system is 576,380, at which time the total system profit attains the peak level of the centralised mode. The aggregate profit of the DCCTSC rose by approximately 6.45 % compared to the scenario without the use of option contracts. In comparison to the decentralised mode prior to the adoption of the revenue-sharing option contract, the profit of the forwarder went up by roughly 5.50 %, the profit of the carrier ascended by roughly 1.78 %, and the profit of the total system climbed by roughly 2.86 %.

Table 6
Empty container decisions and profits for the forwarder, carrier and whole system in different environments.

Different environments	Optimal empty container decision		Profit		
	q_f	q_t	Π_f	Π_t	Π_c
Without option contract	416.89	959.61	10,086	531,390	541,476
With option contract (centralised mode)	752.38	793.47	576,380		
With option contract (decentralised mode)	980.08	755.38	162,320	398,060	560,380

9. Conclusion

In the context of stochastic market demand, this study investigates the problem of empty container capacity planning and channel coordination in a dual-channel system composed of members such as the container leasing company, the carrier, and the forwarder, based on option contracts. Firstly, we discussed the scenario where an option contract was not considered and used it as a benchmark model. We then introduced option contracts from the forwarder's perspective and allowed the forwarder to adjust the volume of containers leased before the market demand was updated, thus mitigating the imbalance between the supply and demand of empty containers for the forwarder. In the case of considering the option contract, we sequentially established models for both decentralised and centralised modes, analysed the optimal empty container leasing decisions of the carrier and forwarder in the different modes, and determined the peak level of channel coordination that can be achieved. Building upon this, we further introduced an improved revenue-sharing contract to coordinate this DCCTSC system, gave a relational equation for the contract parameters to be satisfied when the system is coordinated, and analysed the conditions that the relevant parameters should satisfy when Pareto improvements are realised in the profits of the two members.

The numerical analysis results show that the total profit for the entire DCCTSC is the lowest when the option contract is not taken into account.

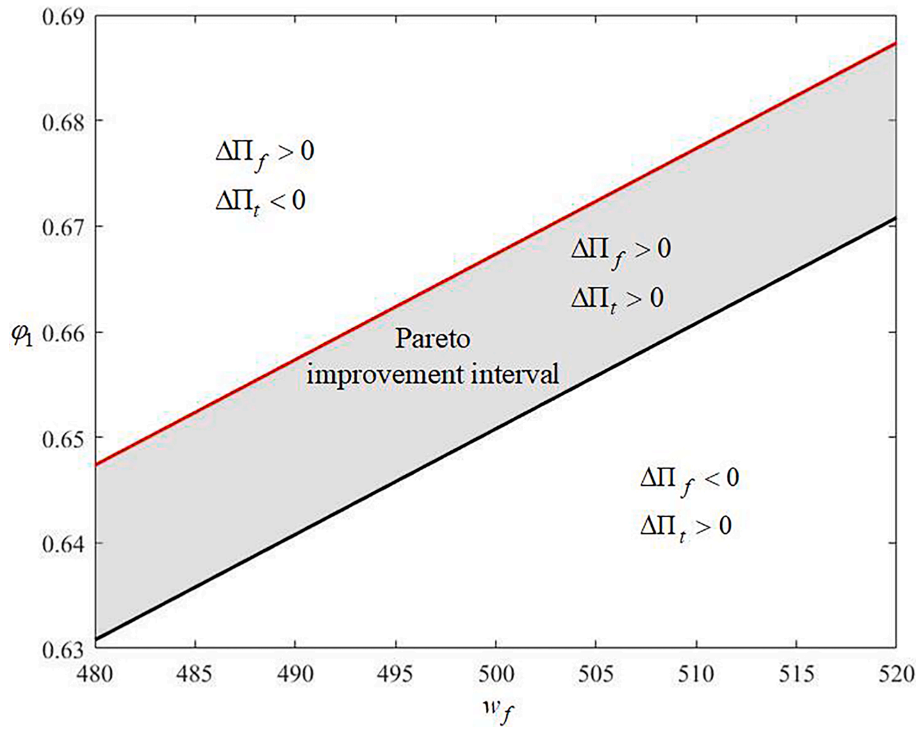

Fig. 12. The Pareto improvement interval illustration for the carrier and the forwarder.

When considering the option contract, the forwarder's exposure to risk due to uncertainty in demand is reduced, and its profits rise significantly. Still, the carrier suffers a loss of profits as a result. Therefore, the aggregate earnings for the carrier and forwarder under the decentralised decision-making scenario is not as high as under the centralised decision-making scenario, but is still higher than when the option contract is not considered. When the revenue-sharing contract is further incorporated based on considering the option contract, by setting reasonable revenue sharing ratios as well as option contract parameters, the losses of the carrier can be compensated. The DCCTSC can be perfectly coordinated, the overall profit of the system can attain the level of the centralised decision-making scenario, and the carrier and forwarder can attain a mutually advantageous outcome. The modification of parameters will merely alter the profit-sharing among participants, without affecting the coordination of this dual-channel system.

Aiming at the perishability of container transport service, we apply option theory to container transportation service system and design an empty container option leasing mechanism between the carrier and the forwarder. This mitigates the risk of empty container ordering by the forwarder and broadens the application field of option theory. We concurrently consider both direct canvassing channels and traditional canvassing channels, and study the coordination problem of empty container decision-making in a dual-channel system where there is a complex competitive relationship between carriers and forwarders, which enriches the theoretical research framework of dual-channel in the shipping chain. We addressed the optimisation problem of empty container resources through a decentralised external decision-making approach, aligning more closely with practical scenarios. By incorporating options into the design of the empty container coordination mechanism, we design a coordination mechanism tailored to the characteristics of the container transport industry. This enhanced the flexibility of traditional coordination mechanisms, effectively fostering collaboration among the main entities in the container transportation service chain.

The aforementioned research provides valuable managerial insights for the forwarder and the carrier in the DCCTSC in terms of empty container decision-making:

(1) The forwarder in the DCCTSC does not have to order too many empty containers to hedge against the risks arising from market demand uncertainties in the traditional channel. Instead, it can enhance its revenue by signing an option contract with the carrier, which minimises the costs associated with empty container procurement and storage.
(2) A mutually advantageous outcome can be attained through contractual coordination between the carrier and the forwarder in a DCCTSC. However, when members use the option contract to reduce the ordering risk faced by the forwarder, the carrier's profits are damaged, preventing both the carrier and forwarder from attaining a win–win outcome. Hence, considering the option contract alone does not optimise profits for both the forwarder and the carrier. To offset for the loss of the carrier, the introduction of a revenue-sharing contract is further proposed. In this arrangement, the carrier and the forwarder negotiate a set of equitable revenue sharing ratios and provide each other with reasonable compensation This strategy enables both parties to share risks and rewards, facilitating a perfect win–win coordination within the dual-channel system. It effectively addresses the limitations of a single option contract coordination and provides a theoretical foundation for alliances between the carrier and the forwarder in the DCCTSC.
(3) The correspondence between option contract parameters and revenue-sharing contract parameters allows the forwarder is able to negotiate parameter values with the carrier according to their individual circumstances. For example, if the forwarder operates on a smaller scale and lacks the necessary capital to secure a sufficient number of empty containers in advance, it can engage in negotiations with the carrier. This negotiation might involve cutting the forwarder's profit share percentage in the traditional channel and growing the carrier's profit share percentage in the direct channel. As a consequence, more revenue is allocated to the carrier in exchange for lower prices for option orders and execution, thereby alleviating initial financial pressure.

Subsequent research holds several avenues for further exploration.

First, the research of this paper is based on the shipping market in a stable state. Still, in real life, there is a distinction between the slack season market and peak season market in the shipping market. So future research can be conducted for different scenarios of the shipping market. Second, the article does not consider the case of forwarder ordering from the spot market or carrier ordering from the option contract market. The next step could consider a more complex hybrid market consisting of the option contract market and the spot market and compare it with the option contract market alone to draw more valuable conclusions. Finally, this paper only considers the empty container decisions of the members of the DCCTSC in a single sales cycle. Future research could extend to multi-cycle scenarios, exploring the advantages that option contracts bring to carriers, forwarders, and the entire dual-channel system under multi-cycle models.

CRediT authorship contribution statement

Tian Luo: Conceptualization, Formal analysis, Investigation, Methodology, Resources, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **Daofang Chang:** Funding acquisition, Project administration, Supervision.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A

Proof of Proposition 1. Equations (1) and (2) can be explicitly expanded as below:

$$
\begin{aligned}
\Pi_f^0\left(q_f^0,q_t^0\right) = p_f\Bigg\{ & \int_0^{q_f^0}\int_{q_t^0}^{-\frac{1}{\lambda_t}x+\frac{q_f^0}{\lambda_t}+q_t^0}\left[x+\lambda_t\left(y-q_t^0\right)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& +\int_{q_f^0}^{\infty}\int_0^{\infty} q_f^0 f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_0^{q_f^0}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^0}{\lambda_t}+q_t^0}^{\infty} q_f^0 f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& +\int_0^{q_f^0}\int_0^{q_t^0} xf(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} - wq_f^0 - g_f\Bigg\{\int_{q_f^0}^{\infty}\int_0^{q_t^0}\left(x-q_f^0\right)f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& +\int_{q_f^0}^{\infty}\int_{q_t^0}^{\infty}\left[x+\lambda_t\left(y-q_t^0\right)-q_f^0\right]f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& +\int_0^{q_f^0}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^0}{\lambda_t}+q_t^0}^{\infty}\left[x+\lambda_t\left(y-q_t^0\right)-q_f^0\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} \\
& -h_f\Bigg\{\int_0^{q_f^0}\int_0^{q_t^0}\left(q_f^0-x\right)f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& +\int_0^{q_f^0}\int_{q_t^0}^{-\frac{1}{\lambda_t}x+\frac{q_f^0}{\lambda_t}+q_t^0}\left[q_f^0-x-\lambda_t\left(y-q_t^0\right)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\}
\end{aligned} \tag{A.1}
$$

$$
\begin{aligned}
\Pi_t^0\left(q_f^0, q_t^0\right) = {} & p_t\Bigg\{ \int_0^{q_f^0}\int_0^{q_t^0} y f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{\infty}\int_{q_t^0}^{\infty} q_t^0 f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f^0+q_t^0}^{q_t^0} q_t^0 f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^0+\frac{q_t^0}{\lambda_f}}^{\infty}\int_0^{q_t^0} q_t^0 f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f^0+q_t^0} \left[y+\lambda_f\left(x-q_f^0\right)\right] f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg\} \\
& - g_t\Bigg\{ \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f^0+q_t^0}^{q_t^0} \left[y+\lambda_f\left(x-q_f^0\right)-q_t^0\right] f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f^0+\frac{q_t^0}{\lambda_f}}^{\infty}\int_0^{q_t^0} \left[y+\lambda_f\left(x-q_f^0\right)-q_t^0\right] f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f^0}^{\infty}\int_{q_t^0}^{\infty} \left[y+\lambda_f\left(x-q_f^0\right)-q_t^0\right] f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^0}\int_{q_t^0}^{\infty} \left(y-q_t^0\right) f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg\} \\
& - h_t\Bigg\{ \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f^0+q_t^0} \left[q_t^0-y-\lambda_f\left(x-q_f^0\right)\right] f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_0^{q_f^0}\int_0^{q_t^0} \left(q_t^0-y\right) f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg\} + w q_f^0 - c_f q_f^0 - c_t q_t^0
\end{aligned} \tag{A.2}
$$

The derivative of $\Pi_t^0\left(q_f^0, q_t^0\right)$ with respect to q_t^0 can be formulated as:

$$
\frac{d\Pi_t^0\left(q_f^0, q_t^0\right)}{dq_t^0} = \frac{\partial\Pi_t^0\left(q_f^0, q_t^0\right)}{\partial q_f^0}\frac{dq_f^0}{dq_t^0} + \frac{\partial\Pi_t^0\left(q_f^0, q_t^0\right)}{\partial q_t^0} \tag{A.3}
$$

Among which,

$$
\begin{aligned}
\frac{\partial\Pi_t^0\left(q_f^0, q_t^0\right)}{\partial q_f^0} = {} & -\lambda_f p_t \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f^0+q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \lambda_f g_t\Bigg[\int_{q_f^0+\frac{q_t^0}{\lambda_f}}^{\infty}\int_0^{q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f^0+q_t^0}^{q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f^0}^{\infty}\int_{q_t^0}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg] - \lambda_f h_t \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f^0+q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x + w - c_f
\end{aligned} \tag{A.4}
$$

$$
\begin{aligned}
\frac{\partial\Pi_t^0\left(q_f^0, q_t^0\right)}{\partial q_t^0} = {} & (p_t+g_t)\Bigg[\int_0^{\infty}\int_{q_t^0}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^0+\frac{q_t^0}{\lambda_f}}^{\infty}\int_0^{q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f^0+q_t^0}^{q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg] - h_t\Bigg[\int_0^{q_f^0}\int_0^{q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f^0+q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg] - c_t
\end{aligned} \tag{A.5}
$$

$$\begin{aligned}
\therefore \frac{d\Pi_t^0\left(q_f^0, q_t^0\right)}{dq_t^0} = & \left\{ -\lambda_f p_t \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}} \int_0^{-\lambda_f x+\lambda_f q_f^0+q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x + w - c_f \right. \\
& +\lambda_f g_t \left[\int_{q_f^0+\frac{q_t^0}{\lambda_f}}^{\infty} \int_0^{q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}} \int_{-\lambda_f x+\lambda_f q_f^0+q_t^0}^{q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \right. \\
& \left. \left. + \int_{q_f^0}^{\infty} \int_{q_t^0}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x \right] - \lambda_f h_t \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}} \int_0^{-\lambda_f x+\lambda_f q_f^0+q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \right\} \cdot \frac{dq_f^0}{dq_t^0} \\
& + \left\{ (p_t + g_t) \left[\int_0^{\infty} \int_{q_t^0}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^0+\frac{q_t^0}{\lambda_f}}^{\infty} \int_0^{q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \right. \right. \\
& \left. + \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}} \int_{-\lambda_f x+\lambda_f q_f^0+q_t^0}^{q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \right] - h_t \left[\int_0^{q_f^0} \int_0^{q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \right. \\
& \left. \left. + \int_{q_f^0}^{q_f^0+\frac{q_t^0}{\lambda_f}} \int_0^{-\lambda_f x+\lambda_f q_f^0+q_t^0} f(x)g(y)\mathrm{d}y\mathrm{d}x \right] - c_t \right\}
\end{aligned} \tag{A.6}$$

$$\therefore \left.\frac{d\Pi_t^0\left(q_f^0, q_t^0\right)}{dq_t^0}\right|_{q_f^0=0} = p_t + g_t - c_t > 0 \left.\frac{d\Pi_t^0\left(q_f^0, q_t^0\right)}{dq_t^0}\right|_{q_f^0=\infty} = -h_t - c_t < 0$$

Due to the continuity of the function $\frac{d\Pi_t^0\left(q_f^0, q_t^0\right)}{dq_t^0}$, in conjunction with the zero-point theorem, there is at least one solution that results in $\frac{d\Pi_t^0\left(q_f^0, q_t^0\right)}{dq_t^0} = 0$. Let $S0$ represents the set of q_t^0 that satisfies the condition $\frac{d\Pi_t^0\left(q_f^0, q_t^0\right)}{dq_t^0} = 0$. Hence, there exists a value $q_t^{0*} = \underset{q_t^0 \in S0}{\arg} \max \Pi_t^0\left(q_t^0\right)$ such that under a traditional wholesale price contract, there exists an optimal set of empty container leasing quantities for both the forwarder and the carrier, denoted as $\left(q_f^{0*}, q_t^{0*}\right)$.

Proof of Proposition 2. We can express $\Pi_f\left(q_f, q_t\right)$ and $\Pi_t\left(q_f, q_t\right)$ in the following manner:

$$\begin{aligned}
\Pi_f\left(q_f, q_t\right) = & \left(p_f - w_f\right) \left\{ \int_0^{q_f} \int_0^{q_t} x f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f} \int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x \right. \\
& \left. + \int_{q_f}^{\infty} \int_0^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f} \int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t} \left[x + \lambda_t (y - q_t)\right] f(x)g(y)\mathrm{d}y\mathrm{d}x \right\} \\
& - g_f \left\{ \int_{q_f}^{\infty} \int_0^{q_t} \left(x - q_f\right) f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{\infty} \int_{q_t}^{\infty} \left[x + \lambda_t (y - q_t) - q_f\right] f(x)g(y)\mathrm{d}y\mathrm{d}x \right. \\
& \left. + \int_0^{q_f} \int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty} \left[x + \lambda_t (y - q_t) - q_f\right] f(x)g(y)\mathrm{d}y\mathrm{d}x \right\} - v_f q_f
\end{aligned} \tag{A.7}$$

$$
\begin{aligned}
\Pi_t\left(q_f, q_t\right) = p_t\Bigg\{ & \int_0^{q_f}\int_0^{q_t} y f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t}^{q_t} q_t f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t}\left[y+\lambda_f\left(x-q_f\right)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{\infty}\int_{q_t}^{\infty} q_t f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t} q_t f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} + w_f\Bigg\{\int_0^{q_f}\int_0^{q_t} x f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{\infty}\int_0^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}\left[x+\lambda_t(y-q_t)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} \\
& - g_t\Bigg\{\int_0^{q_f}\int_{q_t}^{\infty}(y-q_t)f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{\infty}\int_{q_t}^{\infty}\left[y+\lambda_f\left(x-q_f\right)-q_t\right]f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t}^{q_t}\left[y+\lambda_f\left(x-q_f\right)-q_t\right]f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t}\left[y+\lambda_f\left(x-q_f\right)-q_t\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} - h_t\Bigg\{\int_0^{q_f}\int_0^{q_t}(q_t-y)f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t}\left[q_t-y-\lambda_f\left(x-q_f\right)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} \\
& - h_f\Bigg\{\int_0^{q_f}\int_0^{q_t}(q_f-x)f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + \int_0^{q_f}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}\left[q_f-x-\lambda_t(y-q_t)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} + v_f q_f - c_t q_t - c_f q_f
\end{aligned}
\tag{A.8}
$$

The derivative of $\Pi_t\left(q_f, q_t\right)$ with respect to q_t can be formulated as:

$$
\frac{d\Pi_t\left(q_f, q_t\right)}{dq_t} = \frac{\partial\Pi_t\left(q_f, q_t\right)}{\partial q_f}\frac{dq_f}{dq_t} + \frac{\partial\Pi_t\left(q_f, q_t\right)}{\partial q_t}
\tag{A.9}
$$

Among the above equation,

$$
\begin{aligned}
\frac{\partial\Pi_t\left(q_f, q_t\right)}{\partial q_f} = & -\lambda_f p_t\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f(q_t)+q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& + w_f\left[\int_0^{q_f(q_t)}\int_{-\frac{1}{\lambda_t}x+\frac{q_f(q_t)}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f(q_t)}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right] \\
& + \lambda_f g_t\left[\int_{q_f(q_t)+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f(q_t)+q_t}^{q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x\right. \\
& \left. + \int_{q_f(q_t)}^{\infty}\int_{q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right] - h_f\int_0^{q_f(q_t)}\int_0^{-\frac{1}{\lambda_t}x+\frac{q_f(q_t)}{\lambda_t}+q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& - \lambda_f h_t\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f(q_t)+q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x + v_f - c_f
\end{aligned}
\tag{A.10}
$$

$$\frac{\partial\Pi_t(q_f,q_t)}{\partial q_t}=(p_t+g_t)\left[\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f(q_t)+q_t}^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_0^\infty\int_{q_t}^\infty f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f(q_t)+\frac{q_t}{\lambda_f}}^\infty\int_0^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\right]$$
$$-\lambda_t(w_f+h_f)\int_0^{q_f(q_t)}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f(q_t)}{\lambda_t}+q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x$$
$$-h_t\left[\int_0^{q_f(q_t)}\int_0^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f(q_t)+q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\right]-c_t \tag{A.11}$$

$$\therefore\frac{d\Pi_t(q_f,q_t)}{dq_t}=\frac{\partial\Pi_t(q_f,q_t)}{\partial q_f}\frac{dq_f}{dq_t}+\frac{\partial\Pi_t(q_f,q_t)}{\partial q_t}$$
$$=\left\{-\lambda_f p_t\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f(q_t)+q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\right.$$
$$+w_f\left[\int_0^{q_f(q_t)}\int_{-\frac{1}{\lambda_t}x+\frac{q_f(q_t)}{\lambda_t}+q_t}^\infty f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f(q_t)}^\infty\int_0^\infty f(x)g(y)\mathrm{d}y\mathrm{d}x\right]$$
$$+\lambda_f g_t\left[\int_{q_f(q_t)+\frac{q_t}{\lambda_f}}^\infty\int_0^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f(q_t)+q_t}^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\right.$$
$$\left.+\int_{q_f(q_t)}^\infty\int_{q_t}^\infty f(x)g(y)\mathrm{d}y\mathrm{d}x\right]-h_f\int_0^{q_f(q_t)}\int_0^{-\frac{1}{\lambda_t}x+\frac{q_f(q_t)}{\lambda_t}+q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x$$
$$\left.-\lambda_f h_t\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f(q_t)+q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x+v_f-c_f\right\}\cdot\frac{dq_f}{dq_t}$$
$$+\left\{(p_t+g_t)\left[\int_0^\infty\int_{q_t}^\infty f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f(q_t)+q_t}^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\right.\right.$$
$$\left.+\int_{q_f(q_t)+\frac{q_t}{\lambda_f}}^\infty\int_0^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\right]-\lambda_t(w_f+h_f)\int_0^{q_f(q_t)}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f(q_t)}{\lambda_t}+q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x$$
$$\left.-h_t\left[\int_0^{q_f(q_t)}\int_0^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f(q_t)+q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\right]-c_t\right\} \tag{A.12}$$
$$\therefore\left.\frac{d\Pi_t(q_f,q_t)}{dq_t}\right|_{q_f=0}=p_t+g_t-c_t>0\left.\frac{d\Pi_t(q_f,q_t)}{dq_t}\right|_{q_f=\infty}=-h_t-c_t<0$$

Due to the continuity of the function $\frac{\mathrm{d}\Pi_t(q_f,q_t)}{\mathrm{d}q_t}$, in conjunction with the zero-point theorem, it follows that at least one solution that results in $\frac{\mathrm{d}\Pi_t(q_f,q_t)}{\mathrm{d}q_t}=0$. Let S represents the set of q_t that satisfies the condition $\frac{\mathrm{d}\Pi_t(q_f,q_t)}{\mathrm{d}q_t}=0$. Hence, there exists a value $q_t^*=\underset{q_t\in S}{\mathrm{argmax}}\Pi_t(q_t)$ such that both members possess an optimal set of empty container leasing quantities $\left(q_f^*,q_t^*\right)$.

Proof of Corollary 1. We assume:

$$\mu(q_f,q_t)=\frac{\partial\Pi_f(q_f,q_t)}{\partial q_f}$$
$$=(p_f-w_f+g_f)\left[\int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^\infty f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f}^\infty\int_0^\infty f(x)g(y)\mathrm{d}y\mathrm{d}x\right]-v_f \tag{A.13}$$

Applying the implicit differentiation theorem, we find $\frac{dq_f}{dq_t}=-\frac{\partial\mu(q_f,q_t)}{\partial q_t}/\frac{\partial\mu(q_f,q_t)}{\partial q_f}$, among which:

$$\frac{\partial\mu(q_f,q_t)}{\partial q_t}=-(p_f-w_f+g_f)\int_0^{q_f}f(x)g\left(-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t\right)\mathrm{d}x \tag{A.14}$$

$$\frac{\partial\mu(q_f,q_t)}{\partial q_f} = -(p_f - w_f + g_f) \cdot \left[\int_0^{q_t} f(q_f)g(y)\mathrm{d}y + \frac{1}{\lambda_t}\int_0^{q_f} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f}{\lambda_t} + q_t\right)\mathrm{d}x\right] \tag{A.15}$$

So we ascertain:

$$\frac{dq_f}{dq_t} = -\frac{\int_0^{q_f} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f}{\lambda_t} + q_t\right)\mathrm{d}x}{\int_0^{q_t} f(q_f)g(y)\mathrm{d}y + \frac{1}{\lambda_t}\int_0^{q_f} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f}{\lambda_t} + q_t\right)\mathrm{d}x} < 0 \tag{A.16}$$

$$\begin{aligned}\left|\frac{dq_f}{dq_t}\right| &= \frac{\int_0^{q_f} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f}{\lambda_t} + q_t\right)\mathrm{d}x}{\int_0^{q_t} f(q_f)g(y)\mathrm{d}y + \frac{1}{\lambda_t}\int_0^{q_f} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f}{\lambda_t} + q_t\right)\mathrm{d}x} \\ &\leqslant \frac{\int_0^{q_f} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f}{\lambda_t} + q_t\right)\mathrm{d}x}{\frac{1}{\lambda_t}\int_0^{q_f} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f}{\lambda_t} + q_t\right)\mathrm{d}x} = \lambda_t\end{aligned} \tag{A.17}$$

Proof of Corollary 2. From the results above:

$$\frac{\partial\Pi_f}{\partial q_f^*} = (p_f - w_f + g_f) \cdot \left[\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^*}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right] - v_f = 0 \tag{A.18}$$

Both sides of the above equation seek the derivative of v_f at the same time:

$$-(p_f - w_f + g_f) \cdot \left[\int_0^{q_t} f\left(q_f^*\right)g(y)\mathrm{d}y + \frac{1}{\lambda_t}\int_0^{q_f^*} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f^*}{\lambda_t} + q_t\right)\mathrm{d}x\right]\cdot\frac{\partial q_f^*}{\partial v_f} - 1 = 0 \tag{A.19}$$

$$\frac{\partial q_f^*}{\partial v_f} = \frac{-1}{(p_f - w_f + g_f)\left[\int_0^{q_t} f\left(q_f^*\right)g(y)\mathrm{d}y + \frac{1}{\lambda_t}\int_0^{q_f^*} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f^*}{\lambda_t} + q_t\right)\mathrm{d}x\right]} < 0 \tag{A.20}$$

Both sides of the above equation seek the derivative of w_f at the same time:

$$\begin{aligned}&(-1)\cdot\left[\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^*}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right] - (p_f - w_f + g_f) \\ &\quad\cdot\left[\int_0^{q_t} f\left(q_f^*\right)g(y)\mathrm{d}y + \frac{1}{\lambda_t}\int_0^{q_f^*} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f^*}{\lambda_t} + q_t\right)\mathrm{d}x\right]\cdot\frac{\partial q_f^*}{\partial w_f} = 0\end{aligned} \tag{A.21}$$

$$\therefore\frac{\partial q_f^*}{\partial w_f} = -\frac{\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^*}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x}{(p_f - w_f + g_f)\cdot\left[\int_0^{q_t} f\left(q_f^*\right)g(y)\mathrm{d}y + \frac{1}{\lambda_t}\int_0^{q_f^*} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f^*}{\lambda_t} + q_t\right)\mathrm{d}x\right]} < 0 \tag{A.22}$$

Both sides of the above equation seek the derivative of the unit canvassing price of the traditional channel at the same time:

$$\begin{aligned}&\left[\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^*}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right] - (p_f - w_f + g_f) \\ &\quad\cdot\left[\int_0^{q_t} f\left(q_f^*\right)g(y)\mathrm{d}y + \frac{1}{\lambda_t}\int_0^{q_f^*} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f^*}{\lambda_t} + q_t\right)\mathrm{d}x\right]\cdot\frac{\partial q_f^*}{\partial p_f} = 0\end{aligned} \tag{A.23}$$

$$\frac{\partial q_f^*}{\partial p_f} = \frac{\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^*}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x}{\left(p_f - w_f + g_f\right)\left[\int_0^{q_t} f\left(q_f^*\right)g(y)\mathrm{d}y + \frac{1}{\lambda_t}\int_0^{q_f^*} f(x)g\left(-\frac{1}{\lambda_t}x + \frac{q_f^*}{\lambda_t} + q_t\right)\mathrm{d}x\right]} > 0 \tag{A.24}$$

Proof of Corollary 3. Find the derivatives of Π_f with respect to v_f, w_f and p_f respectively:

$$\frac{\partial \Pi_f\left(q_f^*\right)}{\partial v_f} = -q_f^* - v_f\cdot\frac{\partial q_f^*}{\partial v_f}$$

$$+\left(p_f - w_f + g_f\right)\left[\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^*}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right]\cdot\frac{\partial q_f^*}{\partial v_f} \tag{A.25}$$

$$= -q_f^* + \frac{\partial \Pi_f\left(q_f^*\right)}{\partial q_f^*}\cdot\frac{\partial q_f^*}{\partial v_f}$$

$$\because \frac{\partial \Pi_f\left(q_f^*\right)}{\partial q_f^*} = 0$$

$$\therefore \frac{\partial \Pi_f\left(q_f^*\right)}{\partial v_f} = -q_f^* < 0$$

$$\frac{\partial \Pi_f\left(q_f^*\right)}{\partial w_f} = -\left\{\int_0^{q_f^*}\int_0^{q_t} xf(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x\right.$$

$$\left. + \int_{q_f^*}^{\infty}\int_0^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t} [x + \lambda_t(y - q_t)]f(x)g(y)\mathrm{d}y\mathrm{d}x\right\} + \frac{\partial q_f^*}{\partial w_f}$$

$$\cdot\left\{\left(p_f - w_f + g_f\right)\left[\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^*}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right] - v_f\right\} \tag{A.26}$$

$$= \frac{\partial \Pi_f\left(q_f^*\right)}{\partial q_f^*}\cdot\frac{\partial q_f^*}{\partial w_f} - \left\{\int_0^{q_f^*}\int_0^{q_t} xf(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x\right.$$

$$\left. + \int_{q_f^*}^{\infty}\int_0^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t} [x + \lambda_t(y - q_t)]f(x)g(y)\mathrm{d}y\mathrm{d}x\right\}$$

$$\because \frac{\partial \Pi_f\left(q_f^*\right)}{\partial q_f^*} = 0$$

$$\therefore \frac{\partial \Pi_f\left(q_f^*\right)}{\partial w_f} = -\left\{\int_0^{q_f^*}\int_0^{q_t} xf(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x\right.$$

$$\left. + \int_{q_f^*}^{\infty}\int_0^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t} [x + \lambda_t(y - q_t)]f(x)g(y)\mathrm{d}y\mathrm{d}x\right\} < 0 \tag{A.27}$$

$$\frac{\partial \Pi_f\left(q_f^*\right)}{\partial p_f} = \left\{ \int_0^{q_f^*}\int_0^{q_t} x f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x \right.$$
$$\left. + \int_{q_f^*}^{\infty}\int_0^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t} [x+\lambda_t(y-q_t)]f(x)g(y)\mathrm{d}y\mathrm{d}x \right\} + \frac{\partial q_f^*}{\partial p_f}$$
$$\cdot\left\{ \left(p_f - w_f + g_f\right)\left[\int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f^*}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right] - v_f \right\} \tag{A.28}$$
$$= \frac{\partial \Pi_f\left(q_f^*\right)}{\partial q_f^*}\cdot\frac{\partial q_f^*}{\partial p_f} + \left\{ \int_0^{q_f^*}\int_0^{q_t} x f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x \right.$$
$$\left. + \int_{q_f^*}^{\infty}\int_0^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t} [x+\lambda_t(y-q_t)]f(x)g(y)\mathrm{d}y\mathrm{d}x \right\}$$

$$\because \frac{\partial \Pi_f\left(q_f^*\right)}{\partial q_f^*} = 0$$

$$\therefore \frac{\partial \Pi_f\left(q_f^*\right)}{\partial p_f} = \int_0^{q_f^*}\int_0^{q_t} x f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t}^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x$$
$$+ \int_{q_f^*}^{\infty}\int_0^{\infty} q_f^* f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f^*}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f^*}{\lambda_t}+q_t} [x+\lambda_t(y-q_t)]f(x)g(y)\mathrm{d}y\mathrm{d}x > 0 \tag{A.29}$$

Find the derivatives of Π_t with respect to v_f, w_f and p_t separately:

$$\frac{\partial \Pi_t\left(q_t^*\right)}{\partial v_f} = \left\{ (p_t + g_t)\left[\int_0^{\infty}\int_{q_t^*}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t^*}^{q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x \right.\right.$$
$$\left. + \int_{q_f+\frac{q_t^*}{\lambda_f}}^{\infty}\int_0^{q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x\right] - \lambda_t\left(w_f + h_f\right)\int_0^{q_f}\int_{q_t^*}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x + q_f$$
$$\left. - h_t\left[\int_0^{q_f}\int_0^{q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x\right] - c_t \right\}\cdot\frac{\partial q_t^*}{\partial v_f} \tag{A.30}$$
$$= \frac{\partial \Pi_t\left(q_t^*\right)}{\partial q_t^*}\cdot\frac{\partial q_t^*}{\partial v_f} + q_f$$

$$\because \frac{\partial \Pi_t\left(q_t^*\right)}{\partial q_t^*} = 0 \therefore \frac{\partial \Pi_t\left(q_t^*\right)}{\partial v_f} = q_f > 0$$

$$\begin{aligned}
\frac{\partial \Pi_t(q_t^*)}{\partial w_f} &= \Bigg\{ \int_0^{q_f}\int_0^{q_t^*} x f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t^*}^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f}^{\infty}\int_0^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f}\int_{q_t^*}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t^*} \left[x+\lambda_t\left(y-q_t^*\right)\right] f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg\} \\
&+ \Bigg\{ (p_t+g_t)\Bigg[\int_0^{\infty}\int_{q_t^*}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t^*}^{q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f+\frac{q_t^*}{\lambda_f}}^{\infty}\int_0^{q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg] - \lambda_t\left(w_f+h_f\right)\int_0^{q_f}\int_{q_t^*}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&- h_t\Bigg[\int_0^{q_f}\int_0^{q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg] - c_t \Bigg\}\cdot\frac{\partial q_t^*}{\partial w_f} \\
&= \int_0^{q_f}\int_0^{q_t^*} x f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f}\int_{q_t^*}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t^*} \left[x+\lambda_t\left(y-q_t^*\right)\right] f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f}^{\infty}\int_0^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t^*}^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x + \frac{\partial \Pi_t(q_t^*)}{\partial q_t^*}\cdot\frac{\partial q_t^*}{\partial w_f}
\end{aligned} \tag{A.31}$$

$$\because \frac{\partial \Pi_t(q_t^*)}{\partial q_t^*} = 0$$

$$\begin{aligned}
\therefore \frac{\partial \Pi_t(q_t^*)}{\partial v_f} &= \int_0^{q_f}\int_0^{q_t^*} x f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t^*}^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f}^{\infty}\int_0^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f}\int_{q_t^*}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t^*} \left[x+\lambda_t\left(y-q_t^*\right)\right] f(x)g(y)\mathrm{d}y\mathrm{d}x > 0
\end{aligned} \tag{A.32}$$

$$\begin{aligned}
\frac{\partial \Pi_t(q_t^*)}{\partial p_t} &= \Bigg\{ \int_0^{q_f}\int_0^{q_t^*} y f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{\infty}\int_{q_t^*}^{\infty} q_t^* f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t^*}^{q_t^*} q_t^* f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f+\frac{q_t^*}{\lambda_f}}^{\infty}\int_0^{q_t^*} q_t^* f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t^*} \left[y+\lambda_f\left(x-q_f\right)\right] f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg\} \\
&+ \Bigg\{ (p_t+g_t)\Bigg[\int_0^{\infty}\int_{q_t^*}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t^*}^{q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f+\frac{q_t^*}{\lambda_f}}^{\infty}\int_0^{q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg] - \lambda_t\left(w_f+h_f\right)\int_0^{q_f}\int_{q_t^*}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&- h_t\Bigg[\int_0^{q_f}\int_0^{q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t^*} f(x)g(y)\mathrm{d}y\mathrm{d}x \Bigg] - c_t \Bigg\}\cdot\frac{\partial q_t^*}{\partial p_t} \\
&= \int_0^{q_f}\int_0^{q_t^*} y f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{\infty}\int_{q_t^*}^{\infty} q_t^* f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t^*}^{q_t^*} q_t^* f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f+\frac{q_t^*}{\lambda_f}}^{\infty}\int_0^{q_t^*} q_t^* f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t^*} \left[y+\lambda_f\left(x-q_f\right)\right] f(x)g(y)\mathrm{d}y\mathrm{d}x + \frac{\partial \Pi_t(q_t^*)}{\partial q_t^*}\cdot\frac{\partial q_t^*}{\partial p_t}
\end{aligned} \tag{A.33}$$

$$\because \frac{\partial \Pi_t(q_t^*)}{\partial q_t^*} = 0$$

$$\begin{aligned}
&\therefore \frac{\partial \Pi_t(q_t^*)}{\partial p_t} = \int_0^{q_f}\int_0^{q_t^*} yf(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{\infty}\int_{q_t^*}^{\infty} q_t^* f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t^*}^{q_t^*} q_t^* f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f+\frac{q_t^*}{\lambda_f}}^{\infty}\int_0^{q_t^*} q_t^* f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&\quad + \int_{q_f}^{q_f+\frac{q_t^*}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t^*} \left[y+\lambda_f\left(x-q_f\right)\right] f(x)g(y)\mathrm{d}y\mathrm{d}x > 0
\end{aligned} \tag{A.34}$$

Proof of Proposition 3. The total system profit function for the DCCTST can be specifically expanded as:

$$\begin{aligned}
&\Pi_c(q_f,q_t) = p_f\Bigg\{\int_0^{q_f}\int_0^{q_t} xf(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f}^{\infty}\int_0^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t} [x+\lambda_t(y-q_t)] f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} \\
&-g_f\Bigg\{\int_{q_f}^{\infty}\int_0^{q_t}(x-q_f)f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{\infty}\int_{q_t}^{\infty}\left[x+\lambda_t(y-q_t)-q_f\right]f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&\quad + \int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty}\left[x+\lambda_t(y-q_t)-q_f\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} + p_t \\
&\cdot\Bigg\{\int_0^{q_f}\int_0^{q_t} yf(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t}\left[y+\lambda_f\left(x-q_f\right)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&\quad + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t}^{q_t} q_t f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t} q_t f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&\quad + \int_0^{\infty}\int_{q_t}^{\infty} q_t f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} - g_t\Bigg\{\int_0^{q_f}\int_{q_t}^{\infty}(y-q_t)f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&\quad + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t}^{q_t}\left[y+\lambda_f\left(x-q_f\right)-q_t\right]f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&\quad + \int_{q_f+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t}\left[y+\lambda_f\left(x-q_f\right)-q_t\right]f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&+ \int_{q_f}^{\infty}\int_{q_t}^{\infty}\left[y+\lambda_f\left(x-q_f\right)-q_t\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} - h_t\Bigg\{\int_0^{q_f}\int_0^{q_t}(q_t-y)f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&\quad + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t}\left[q_t-y-\lambda_f\left(x-q_f\right)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} \\
&\quad -h_f\Bigg\{\int_0^{q_f}\int_0^{q_t}(q_f-x)f(x)g(y)\mathrm{d}y\mathrm{d}x \\
&\quad + \int_0^{q_f}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}\left[q_f-x-\lambda_t(y-q_t)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\Bigg\} - c_t q_t - c_f q_f
\end{aligned} \tag{A.35}$$

The second derivatives are derived as:

$$\begin{aligned}
&\frac{\partial^2 \Pi_c(q_f,q_t)}{\partial q_f^2} = -\lambda_f g_t f(q_f) - (p_t+g_t+h_t) \\
&\cdot\left[\lambda_f^2\int_{q_f}^{q_f+\frac{q_t}{\lambda_f}} f(x)g\left(-\lambda_f x+\lambda_f q_f+q_t\right)\mathrm{d}x - \lambda_f\int_0^{q_t} f\left(q_f\right)g(y)\mathrm{d}x\right] \\
&-\left(p_f+g_f+h_f\right)\left[\int_0^{q_t} f\left(q_f\right)g(y)\mathrm{d}x + \frac{1}{\lambda_t}\int_0^{q_f} f(x)g\left(-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t\right)\mathrm{d}x\right]
\end{aligned} \tag{A.36}$$

$\because p_f+g_f+h_f > \lambda_f(p_t+g_t+h_t)$

$$\therefore (p_f + g_f + h_f) \int_0^{q_t} f(q_f) g(y) \mathrm{d}x > \lambda_f (p_t + g_t + h_t) \int_0^{q_t} f(q_f) g(y) \mathrm{d}x$$

$$\therefore - (p_f + g_f + h_f) \int_0^{q_t} f(q_f) g(y) \mathrm{d}x + \lambda_f (p_t + g_t + h_t) \int_0^{q_t} f(q_f) g(y) \mathrm{d}x < 0$$

$$\therefore \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_f^2} < 0$$

$$\begin{aligned} \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_t^2} &= -\lambda_t q_f g(q_t) - \lambda_t (p_f + g_f + h_f) \\ &\cdot \left[\int_0^{q_f} f(x) g\left(-\frac{1}{\lambda_t} x + \frac{q_f}{\lambda_t} + q_t \right) \mathrm{d}x - \int_0^{q_f} f(x) g(q_t) \mathrm{d}x \right] \\ -(p_t + g_t + h_t) &\left[\int_0^{q_f} f(x) g(q_t) \mathrm{d}x + \int_{q_f}^{q_f + \frac{q_t}{\lambda_f}} f(x) g(- \lambda_f x + \lambda_f q_f + q_t) \mathrm{d}x \right] \end{aligned} \tag{A.37}$$

$$\because p_t + g_t + h_t > \lambda_t (p_f + g_f + h_f)$$

$$\therefore (p_t + g_t + h_t) \int_0^{q_f} f(x) g(q_t) \mathrm{d}x > \lambda_t (p_f + g_f + h_f) \int_0^{q_f} f(x) g(q_t) \mathrm{d}x$$

$$\therefore - (p_t + g_t + h_t) \int_0^{q_f} f(x) g(q_t) \mathrm{d}x + \lambda_t (p_f + g_f + h_f) \int_0^{q_f} f(x) g(q_t) \mathrm{d}x < 0$$

$$\therefore \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_t^2} < 0$$

$$\begin{aligned} \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_f \partial q_t} = \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_t \partial q_f} &= -(p_f + g_f + h_f) \int_0^{q_f} f(x) g\left(-\frac{1}{\lambda_t} x + \frac{q_f}{\lambda_t} + q_t \right) \mathrm{d}x \\ -\lambda_f (p_t + g_t + h_t) &\int_{q_f}^{q_f + \frac{q_t}{\lambda_f}} f(x) g(- \lambda_f x + \lambda_f q_f + q_t) \mathrm{d}x < 0 \end{aligned} \tag{A.38}$$

$$\because \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_f^2} < \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_f \partial q_t} < 0 \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_t^2} < \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_t \partial q_f} < 0$$

$$\begin{aligned} \therefore |H(q_f, q_t)| &= \begin{vmatrix} \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_f^2} & \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_f \partial q_t} \\ \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_t \partial q_f} & \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_t^2} \end{vmatrix} \\ = \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_f^2} \cdot \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_t^2} &- \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_f \partial q_t} \cdot \frac{\partial^2 \Pi_c (q_f, q_t)}{\partial q_t \partial q_f} > 0 \end{aligned} \tag{A.39}$$

$\therefore$ The Hessian matrix is negative definite, and $\Pi_c(q_f, q_t)$ is co-concave on q_f and q_t.

Proof of Proposition 4. We can express Equation (21) in the following manner:

$$\begin{aligned}
\Pi_f^s\left(q_f,q_t\right) = {} & \left(\varphi_1 p_f - w_f\right)\left\{\int_0^{q_f}\int_0^{q_t} xf(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{\infty}\int_0^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x\right. \\
& \left. + \int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty} q_f f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{q_f}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}\left[x+\lambda_t(y-q_t)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\right\} \\
& +(1-\varphi_2)p_t\left\{\int_0^{q_f}\int_0^{q_t} yf(x)g(y)\mathrm{d}y\mathrm{d}x + \int_0^{\infty}\int_{q_t}^{\infty} q_t f(x)g(y)\mathrm{d}y\mathrm{d}x\right. \\
& + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_{-\lambda_f x+\lambda_f q_f+q_t}^{q_t} q_t f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t} q_t f(x)g(y)\mathrm{d}y\mathrm{d}x \\
& \left. + \int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t}\left[y+\lambda_f\left(x-q_f\right)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\right\} - v_f q_f \\
& -g_f\left\{\int_{q_f}^{\infty}\int_0^{q_t}\left(x-q_f\right)f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{\infty}\int_{q_t}^{\infty}\left[x+\lambda_t(y-q_t)-q_f\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\right. \\
& \left. + \int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty}\left[x+\lambda_t(y-q_t)-q_f\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\right\}
\end{aligned} \tag{A.40}$$

Taking the derivatives of $\Pi_f^s\left(q_f,q_t\right)$ yield the following equations:

$$\begin{aligned}
\frac{\partial\Pi_f^s\left(q_f,q_t\right)}{\partial q_f} = {} & -(1-\varphi_2)\lambda_f p_t\int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_f x+\lambda_f q_f+q_t} f(x)g(y)\mathrm{d}y\mathrm{d}x - v_f \\
& \left(\varphi_1 p_f - w_f + g_f\right)\left[\int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x + \int_{q_f}^{\infty}\int_0^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x\right]
\end{aligned} \tag{A.41}$$

$$\begin{aligned}
\frac{\partial^2\Pi_f^s\left(q_f,q_t\right)}{\partial q_f^2} = {} & \left[\varphi_1 p_f - w_f + g_f - (1-\varphi_2)\lambda_f p_t\right]\int_0^{q_t} f\left(q_f\right)g(y)\mathrm{d}y \\
& -\frac{1}{\lambda_t}\left(\varphi_1 p_f - w_f + g_f\right)\int_0^{q_f} f(x)g\left(-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t\right)\mathrm{d}x \\
& -\lambda_f^2(1-\varphi_2)p_t\int_{q_f}^{q_f+\frac{q_t}{\lambda_f}} f(x)g\left(-\lambda_f x+\lambda_f q_f+q_t\right)\mathrm{d}x
\end{aligned} \tag{A.42}$$

Equation (22) can be indicated as:

$$
\begin{aligned}
&\Pi_t^s\left(q_f,q_t\right)=\varphi_2p_t\left\{\int_0^{q_f}\int_0^{q_t}yf(x)g(y)\mathrm{d}y\mathrm{d}x+\int_0^{\infty}\int_{q_t}^{\infty}q_tf(x)g(y)\mathrm{d}y\mathrm{d}x\right.\\
&+\int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_{-\lambda_fx+\lambda_fq_f+q_t}^{q_t}q_tf(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t}q_tf(x)g(y)\mathrm{d}y\mathrm{d}x\\
&\left.+\int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_fx+\lambda_fq_f+q_t}\left[y+\lambda_f\left(x-q_f\right)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\right\}+\left[(1-\varphi_1)p_f+w_f\right]\\
&\cdot\left\{\int_0^{q_f}\int_0^{q_t}xf(x)g(y)\mathrm{d}y\mathrm{d}x+\int_0^{q_f}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}\left[x+\lambda_t(y-q_t)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\right.\\
&\left.+\int_0^{q_f}\int_{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}^{\infty}q_ff(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f}^{\infty}\int_0^{\infty}q_ff(x)g(y)\mathrm{d}y\mathrm{d}x\right\}+v_fq_f\\
&-g_t\left\{\int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_{-\lambda_fx+\lambda_fq_f+q_t}^{q_t}\left[y+\lambda_f\left(x-q_f\right)-q_t\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\right.\\
&+\int_0^{q_f}\int_{q_t}^{\infty}(y-q_t)f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t}\left[y+\lambda_f\left(x-q_f\right)-q_t\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\\
&\left.+\int_{q_f}^{\infty}\int_{q_t}^{\infty}\left[y+\lambda_f\left(x-q_f\right)-q_t\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\right\}-h_t\left\{\int_0^{q_f}\int_0^{q_t}(q_t-y)f(x)g(y)\mathrm{d}y\mathrm{d}x\right.\\
&\left.+\int_{q_f}^{q_f+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_fx+\lambda_fq_f+q_t}\left[q_t-y-\lambda_f\left(x-q_f\right)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\right\}\\
&-h_f\left\{\int_0^{q_f}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f}{\lambda_t}+q_t}\left[q_f-x-\lambda_t(y-q_t)\right]f(x)g(y)\mathrm{d}y\mathrm{d}x\right.\\
&\left.+\int_0^{q_f}\int_0^{q_t}(q_f-x)f(x)g(y)\mathrm{d}y\mathrm{d}x\right\}-c_tq_t-c_fq_f
\end{aligned}\tag{A.43}
$$

Taking the derivative of $\Pi_t^s\left(q_f,q_t\right)$ yield the following equation:

$$
\begin{aligned}
\frac{\partial\Pi_t^s\left(q_f,q_t\right)}{\partial q_t}&=(\varphi_2p_t+g_t)\left[\int_0^{\infty}\int_{q_t}^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f(q_t)+\frac{q_t}{\lambda_f}}^{\infty}\int_0^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\right.\\
&\left.+\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_{-\lambda_fx+\lambda_fq_f(q_t)+q_t}^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\right]\\
&-\lambda_t\left[(1-\varphi_2)p_t+w_f+h_f\right]\int_0^{q_f(q_t)}\int_{q_t}^{-\frac{1}{\lambda_t}x+\frac{q_f(q_t)}{\lambda_t}+q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\\
&-h_t\left[\int_0^{q_f(q_t)}\int_0^{q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x+\int_{q_f(q_t)}^{q_f(q_t)+\frac{q_t}{\lambda_f}}\int_0^{-\lambda_fx+\lambda_fq_f(q_t)+q_t}f(x)g(y)\mathrm{d}y\mathrm{d}x\right]-c_t
\end{aligned}\tag{A.44}
$$

Per the assumptions stipulated for α_1, α_2, β_1, and β_2, we deduce:

$$
\begin{aligned}
\frac{\partial\Pi_c\left(q_f,q_t\right)}{\partial q_f}&=\left(p_f+g_f\right)(1-\alpha_1)-\lambda_f(p_t+h_t)\alpha_2-h_f\alpha_1-c_f\\
&+\lambda_fg_t\left[\int_{q_f}^{\infty}\int_0^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x-\alpha_2\right]=p_f+g_f-c_f\\
&+\lambda_fg_t\int_{q_f}^{\infty}\int_0^{\infty}f(x)g(y)\mathrm{d}y\mathrm{d}x-\left(p_f+g_f+h_f\right)\alpha_1-\lambda_f(p_t+g_t+h_t)\alpha_2\\
&=p_f+g_f-c_f+\lambda_fg_t\int_{q_f}^{\infty}f(x)\mathrm{d}x-\left(p_f+g_f+h_f\right)\alpha_1-\lambda_f(p_t+g_t+h_t)\alpha_2
\end{aligned}\tag{A.45}
$$

$$\begin{aligned}
\frac{\partial \Pi_c(q_f, q_t)}{\partial q_t} &= (p_t + g_t)(1-\beta_1) - \lambda_t(p_f + h_f)\beta_2 - h_t\beta_1 - c_t \\
&+\lambda_t g_f\left[\int_0^{\infty}\int_{q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x - \beta_2\right] = p_t + g_t - c_t \\
&+\lambda_t g_f\int_0^{\infty}\int_{q_t}^{\infty} f(x)g(y)\mathrm{d}y\mathrm{d}x - (p_t + g_t + h_t)\beta_1 - \lambda_t(p_f + g_f + h_f)\beta_2 \\
&= p_t + g_t - c_t + \lambda_t g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y - (p_t + g_t + h_t)\beta_1 - \lambda_t(p_f + g_f + h_f)\beta_2
\end{aligned} \tag{A.46}$$

$$\frac{\partial \Pi_f^s(q_f, q_t)}{\partial q_f} = (\varphi_1 p_f - w_f + g_f)(1-\alpha_1) - \lambda_f(1-\varphi_2)p_t\alpha_2 - v_f \tag{A.47}$$

$$\frac{\partial \Pi_t^s(q_f, q_t)}{\partial q_t} = (\varphi_2 p_t + g_t)\cdot(1-\beta_1) - \lambda_t\left[(1-\varphi_1)p_f + w_f + h_f\right]\beta_2 - h_t\beta_1 - c_t \tag{A.48}$$

Commencing with $\frac{\partial \Pi_c(q_f, q_t)}{\partial q_f} = 0$, we can deduce:

$$\lambda_f = \frac{p_f + g_f - c_f - (p_f + g_f + h_f)\alpha_1}{(p_t + g_t + h_t)\alpha_2 - g_t\int_{q_f}^{\infty} f(x)\mathrm{d}x} \tag{A.49}$$

Commencing with $\frac{\partial \Pi_c(q_f, q_t)}{\partial q_t} = 0$, we can deduce:

$$\lambda_t = \frac{p_t + g_t - c_t - (p_t + g_t + h_t)\beta_1}{(p_f + g_f + h_f)\beta_2 - g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y} \tag{A.50}$$

Commencing with $\frac{\partial \Pi_f^s(q_f, q_t)}{\partial q_f} = 0$, we can deduce:

$$v_f = \lambda_f(1-\varphi_2)p_t\alpha_2 - (\varphi_1 p_f - w_f + g_f)\cdot(1-\alpha_1) \tag{A.51}$$

Commencing with $\frac{\partial \Pi_t^s(q_f, q_t)}{\partial q_t} = 0$, we can deduce:

$$\varphi_2 = \frac{\lambda_t\left[(1-\varphi_1)p_f + w_f + h_f\right]\beta_2 + (g_t + h_t)\beta_1 - g_t + c_t}{p_t(1-\beta_1)} \tag{A.52}$$

Through the combination of equations (A.49) ~ (A.52), we can derive:

$$\begin{aligned}
v_f &= \frac{[p_t + g_t - c_t - (p_t + g_t + h_t)\beta_1]\cdot\left[(\varphi_1 p_f - w_f + g_f)\beta_2 - g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y\right]}{(1-\beta_1)\left[(p_f + g_f + h_f)\beta_2 - g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y\right]} \\
&\cdot\frac{\alpha_2\left[p_f + g_f - c_f - (p_f + g_f + h_f)\alpha_1\right]}{(p_t + g_t + h_t)\alpha_2 - g_t\int_{q_f}^{\infty} f(x)\mathrm{d}x} - (\varphi_1 p_f - w_f + g_f)\cdot(1-\alpha_1)
\end{aligned} \tag{A.53}$$

$$\varphi_2 = \frac{\begin{array}{c}[p_t + g_t - c_t - (p_t + g_t + h_t)\beta_1]\cdot\left[(1-\varphi_1)p_f + w_f + h_f\right]\beta_2 \\ +\left[(p_f + g_f + h_f)\beta_2 - g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y\right]\cdot[(g_t + h_t)\beta_1 - g_t + c_t]\end{array}}{p_t(1-\beta_1)\left[(p_f + g_f + h_f)\beta_2 - g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y\right]} \tag{A.54}$$

Proof of Proposition 5. Replacing the values of φ_2 in Equations (34) and (35), we yield:

$$\begin{aligned}
\Delta\Pi_f &= (\varphi_1 - 1)p_f E\min(q_f, D_f) + E\min(q_t, D_t) \\
&\cdot\frac{[p_t + g_t - c_t - (p_t + g_t + h_t)\beta_1]\cdot\left[(\varphi_1 p_f - w_f + g_f)\beta_2 - g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y\right]}{(1-\beta_1)\left[(p_f + g_f + h_f)\beta_2 - g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y\right]}
\end{aligned} \tag{A.55}$$

$$\begin{aligned}
\Delta\Pi_t &= \frac{[p_t + g_t - c_t - (p_t + g_t + h_t)\beta_1]\cdot\left[g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y - (\varphi_1 p_f - w_f + g_f)\beta_2\right]}{(1-\beta_1)\left[(p_f + g_f + h_f)\beta_2 - g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y\right]} \\
&\cdot E\min(q_t, D_t) + (1-\varphi_1)p_f E\min(q_f, D_f)
\end{aligned} \tag{A.56}$$

Calculating the first-order derivatives of $\Delta\Pi_f$ and $\Delta\Pi_t$ with respect to φ_1 gives:

$$\frac{\partial\Delta\Pi_f}{\partial\varphi_1} = p_f E\min(q_f, D_f) + \frac{[p_t + g_t - c_t - (p_t + g_t + h_t)\beta_1]\cdot p_f\beta_2\cdot E\min(q_t, D_t)}{(1-\beta_1)\left[(p_f + g_f + h_f)\beta_2 - g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y\right]} \tag{A.57}$$

$$\frac{\partial\Delta\Pi_t}{\partial\varphi_1} = \frac{-[p_t + g_t - c_t - (p_t + g_t + h_t)\beta_1]\cdot p_f\beta_2\cdot E\min(q_t, D_t)}{(1-\beta_1)\left[(p_f + g_f + h_f)\beta_2 - g_f\int_{q_t}^{\infty} g(y)\mathrm{d}y\right]} - p_f E\min(q_f, D_f) \tag{A.58}$$

Obviously, $\frac{\partial\Delta\Pi_f}{\partial\varphi_1} = -\frac{\partial\Delta\Pi_t}{\partial\varphi_1}$.

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